

Hybrid kernelised PET reconstruction allowing for negative values

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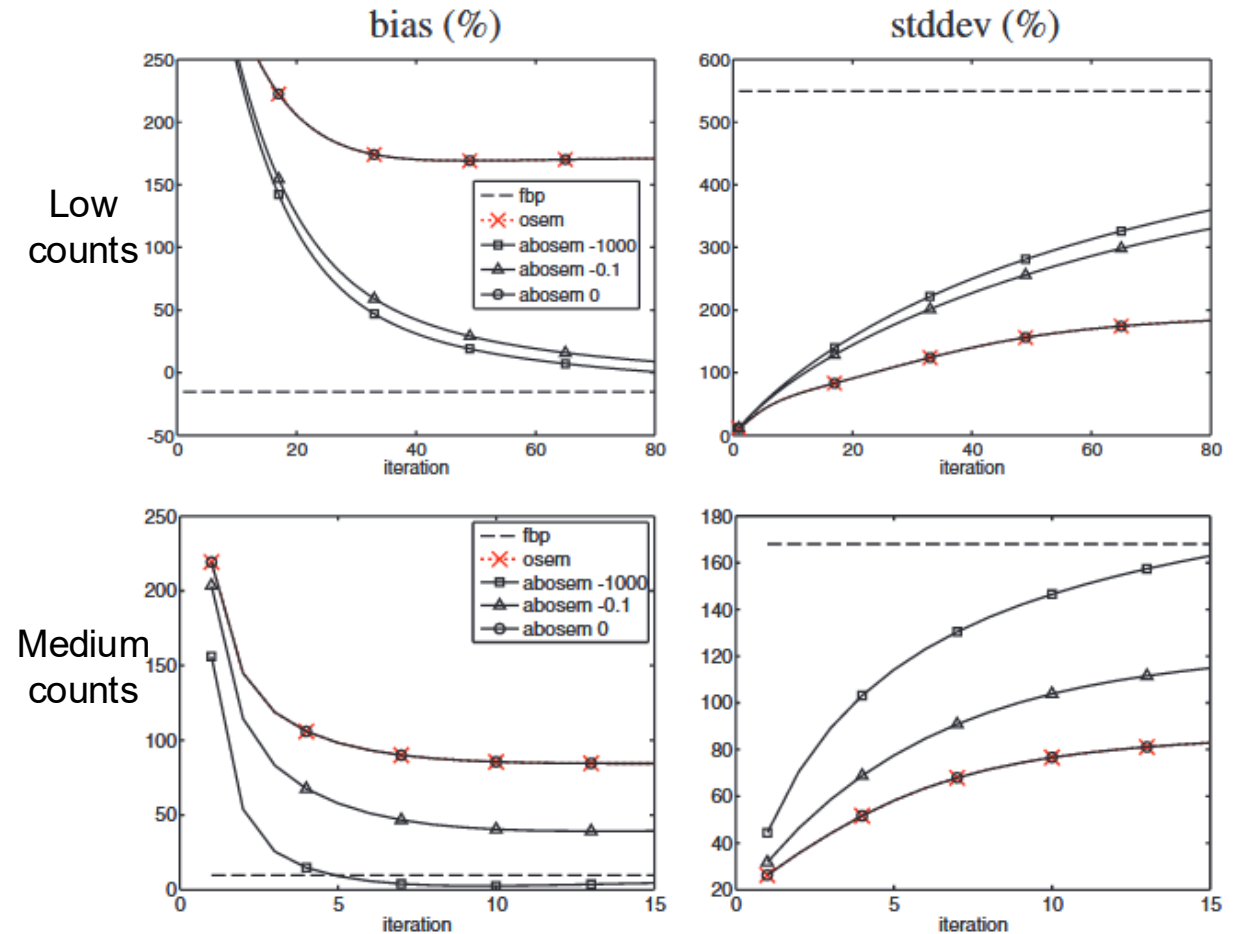
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Background

- Ordered-subset expectation maximisation (OSEM): Positive bias in low counts regions due to the non-negativity constraint.
- AB-OSEM [1,2]: Reduces bias by allowing negative voxel values, but typically increases noise compared with OSEM.



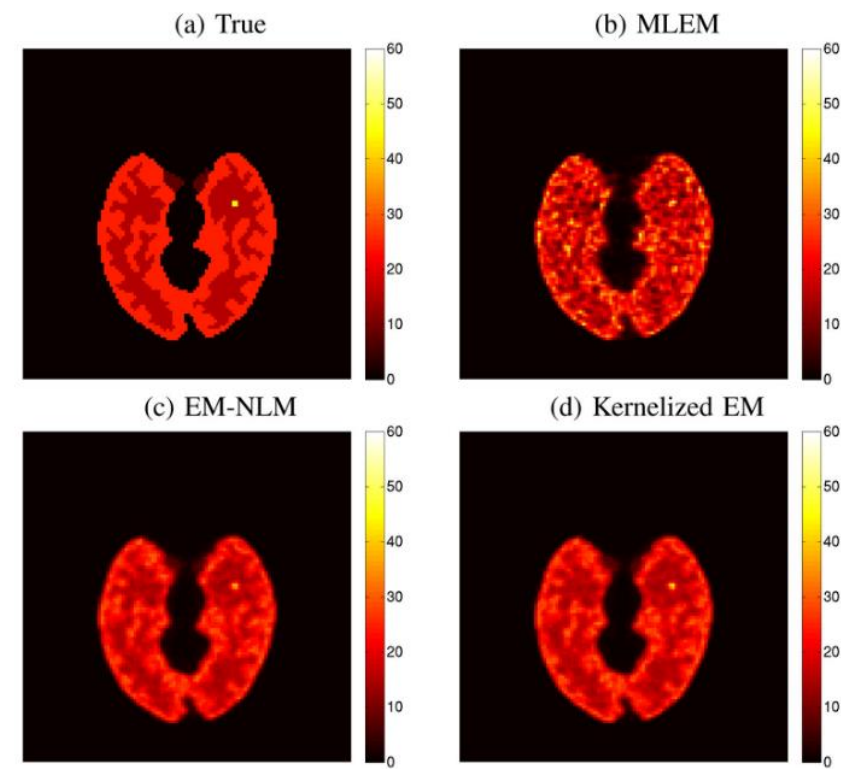
Verhaeghe and Reader, 2010. Phys.Med.Biol.

[1] Byrne, 1998. Inverse Problems. 14 1455

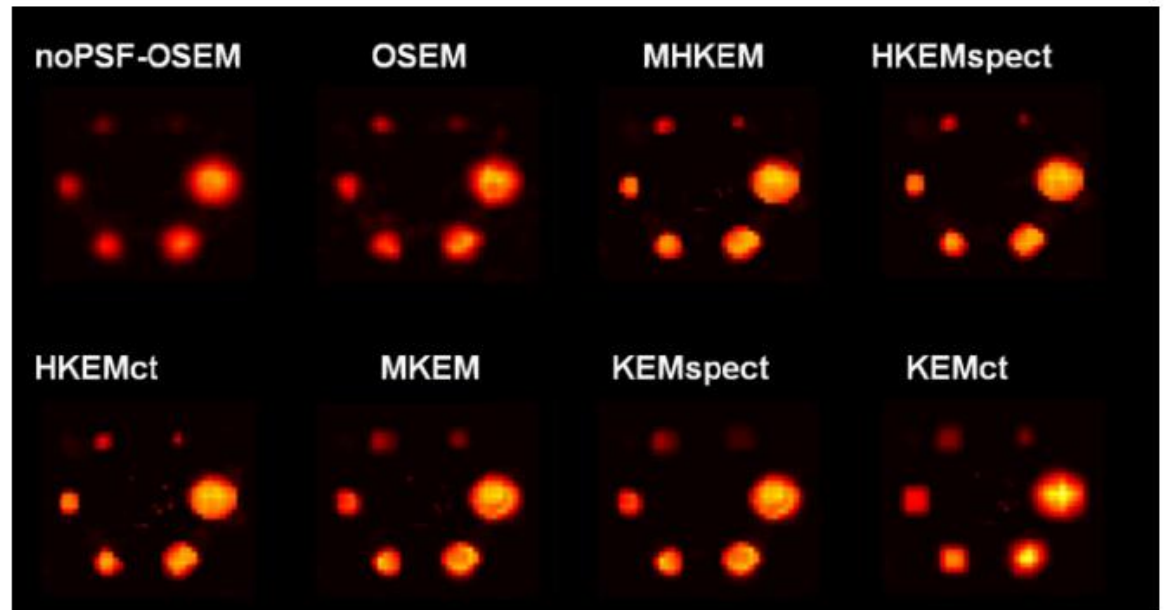
[2] Verhaeghe and Reader, 2010. Phys.Med.Biol. 55 6739

Background

- Kernelised EM (KEM) [1,2]: Incorporates prior information (kernels) to improve PET image quality and quantification.
- HKEM: Extends KEM by combining anatomical priors with PET updates; implemented in STIR [3].



Wang and Qi,
2015. IEEE TMI



[1] Wang and Qi, 2015. IEEE TMI.

[2] Novosad and Reader, 2015. Phys. Med. Biol.

[3] Deidda et al 2019 Inverse Problems

Deidda et al 2019 Inverse Problems

A-OSEM

$$f^{k+1} = \frac{A^k b + B^k a}{A^k + B^k}$$

$$A^k = \frac{f^k - a}{P^T} P^T \frac{y - Pa}{Pf^k + c - Pa}$$

$$B^k = \frac{b - f^k}{P^T} P^T \frac{Pb - y}{Pb - Pf^k + c}$$



$$\hat{f}^{k+1} = \frac{\hat{f}^k - a}{P^T} P^T \frac{y - Pa}{Pf^k + c - Pa} + a$$

P: system matrix

a: lower bound, b upper bound

y: measured projection data

c: additive term

k: iteration number

when taking the higher bound limit $b \rightarrow \infty$.

A-KEM

$$f = K\alpha$$

$$\alpha^{k+1} = \frac{\alpha^k}{K^T P^T} \left(K^T P^T \frac{y}{PK\alpha^k + c} \right)$$



$$f = K\alpha' + a$$

$$\alpha'^{k+1} = \frac{\alpha'^k}{K^T P^T} \left(K^T P^T \frac{y - Pa}{PK\alpha'^k + c} \right)$$

K: kernel matrix

α : coefficient vector to estimate

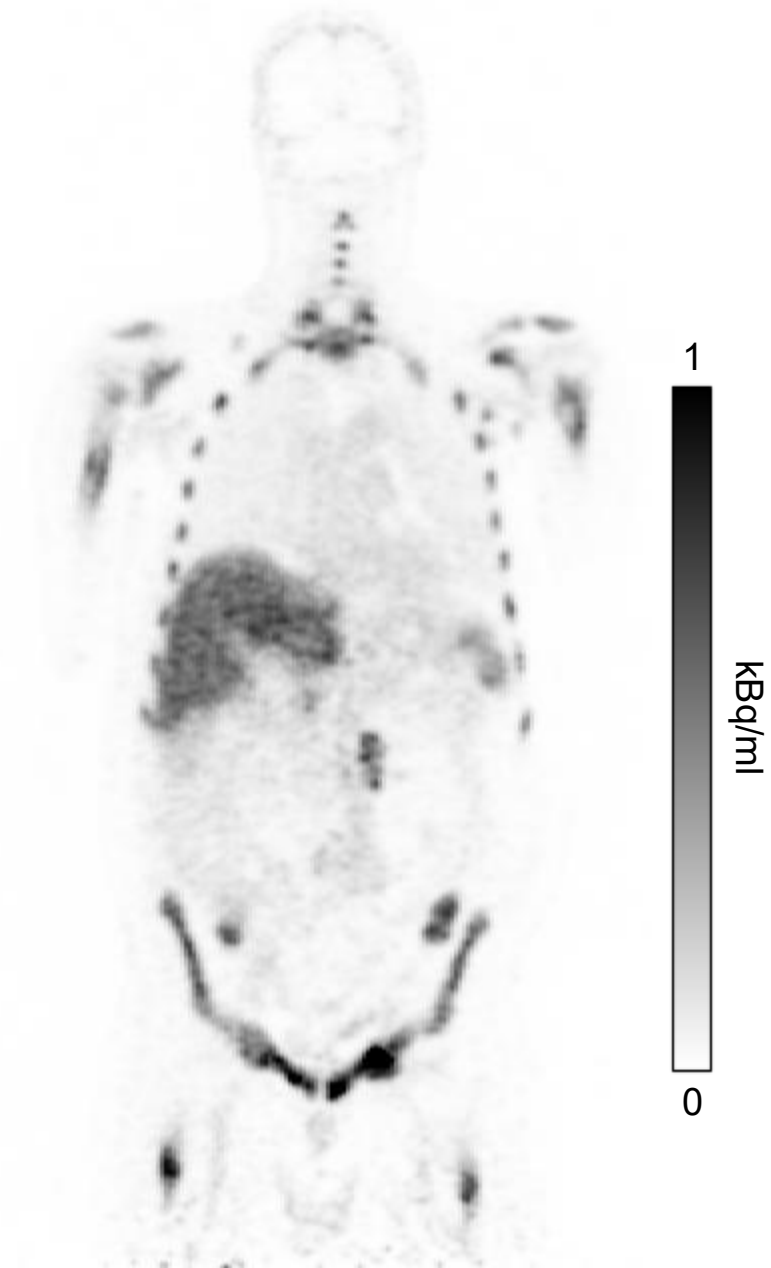
$$K_{CT} = e^{\left(-\frac{\|v_g - v_j\|^2}{2\sigma_c^2}\right)} e^{\left(-\frac{\|x_g - x_j\|^2}{2\sigma_{dc}^2}\right)}$$

$$K_{PET} = e^{\left(-\frac{\|z_g^k - z_j^k\|^2}{2\sigma_p^2}\right)} e^{\left(-\frac{\|x_g - x_j\|^2}{2\sigma_{dp}^2}\right)}$$

The quantity x_j is the coordinate of the j^{th} voxel, z^k is the feature vector that is calculated from the k^{th} PET update image, and σ_c , σ_p , σ_{dc} and σ_{dp} are scaling parameters for the distances.

Patient scan

- ImmunoPET plays an important role in targeted therapy and immunotherapy by imaging intact monoclonal antibodies labeled with longer-lived radionuclides, such as ^{89}Zr (3.3 days half-life).
- It typically involves a low injected dose (e.g. 37 MBq) and imaging at late time points (e.g. 7 days) post-injection.



Standard reconstruction
at day 7 p.i. PSFTOF, 4i5s,
5mm Gaussian filter

Acquisition and reconstruction settings



Siemens Biograph Vision Quadra PET/CT: 25-minute acquisition.

Co-registration of PET & CT using AIRPET (Schaefferkoetter, EJNMMI, 2022).

STIR (Software for Tomographic Image Reconstruction) to reconstruct sinograms generated by Siemens e7tools preprocessed for the Quadra.

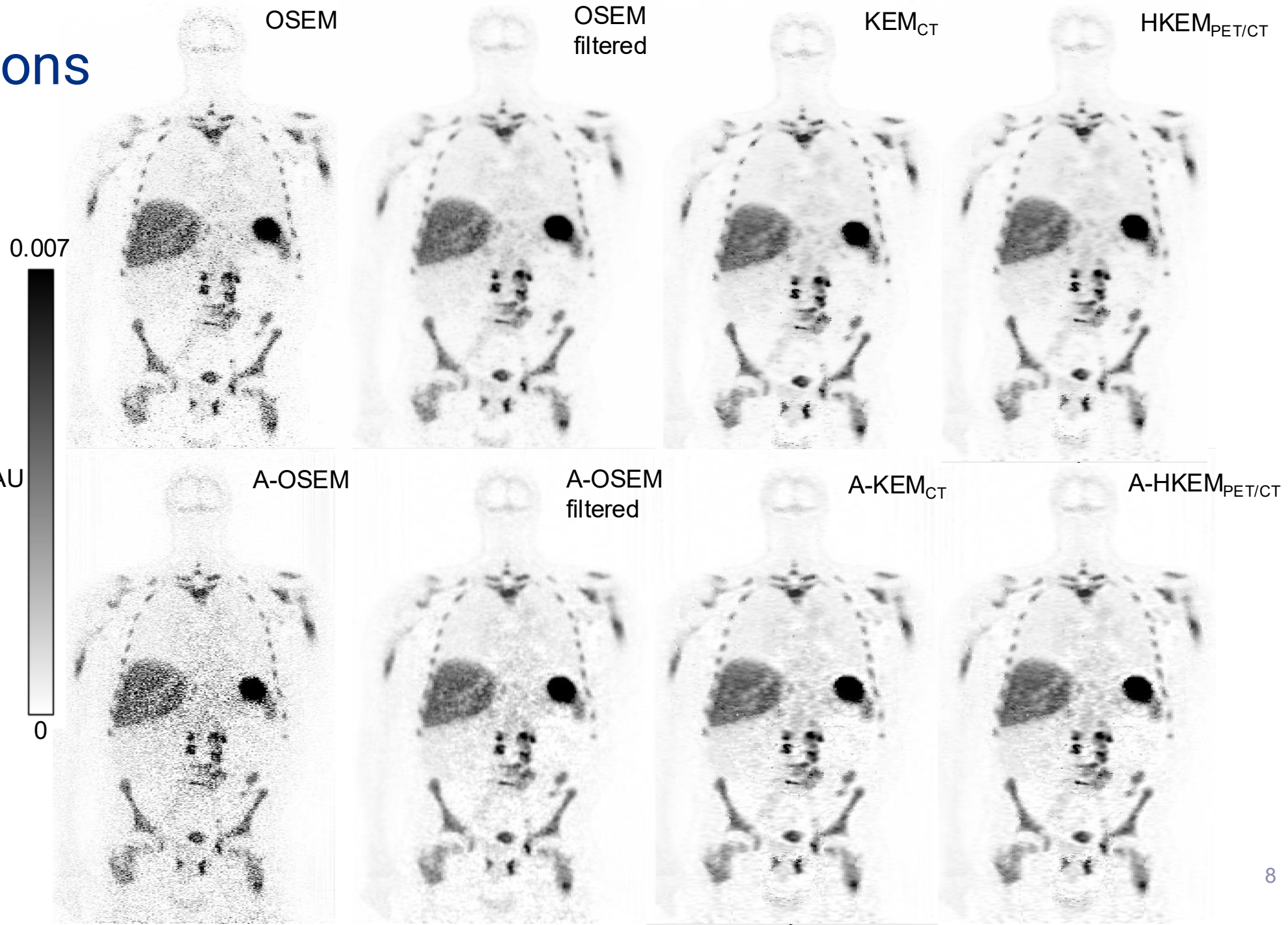
All reconstruction algorithms are implemented within STIR.

Settings: 4 iterations 5 subsets, $220 \times 220 \times 645$, $3.3 \times 3.3 \times 1.645$ mm³, isotropic Gaussian filter: 5mm.

A-OSEM / A-KEM / A-HKEM lower bound: -0.1.

KEMs / HKEMs: Neighbors: $3 \times 3 \times 3$, functional edge iterative PET σ_p : 1, anatomical edge CT σ_c : 1, spatial distance σ_{dp} : 5

Reconstructions

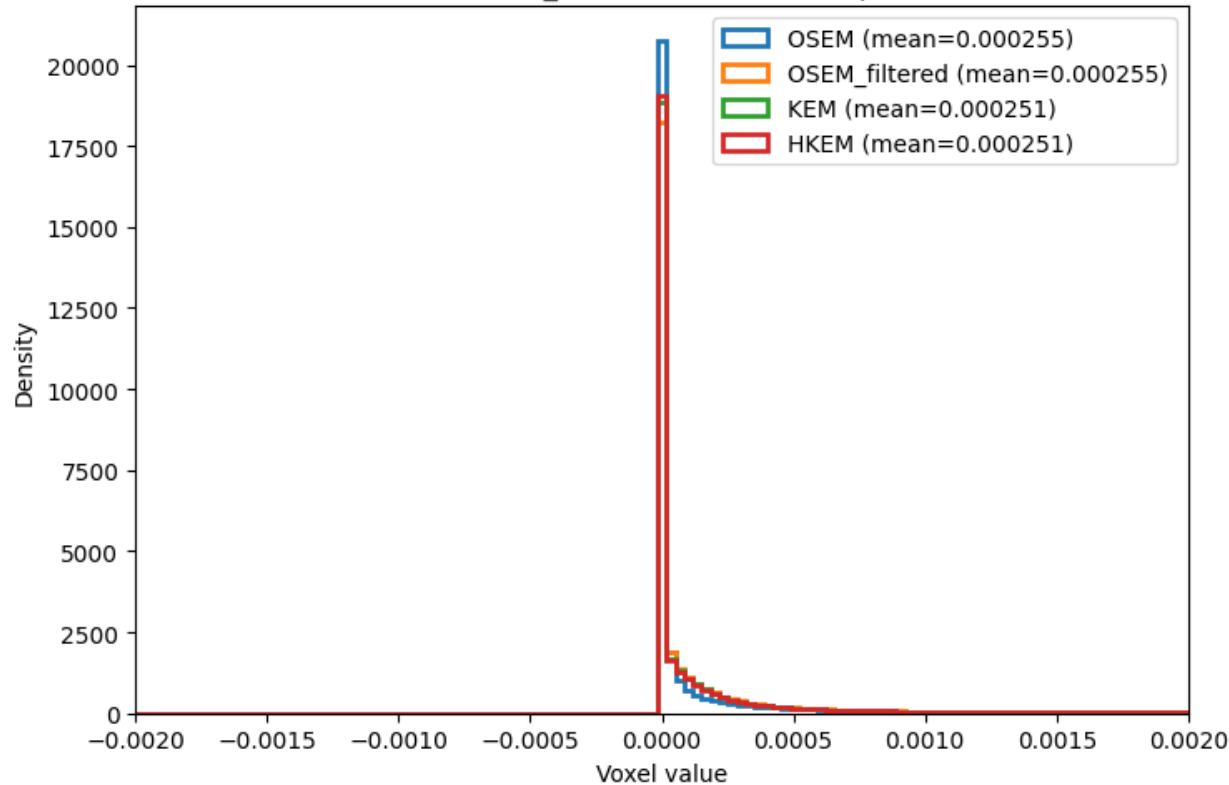


Co-registered CT

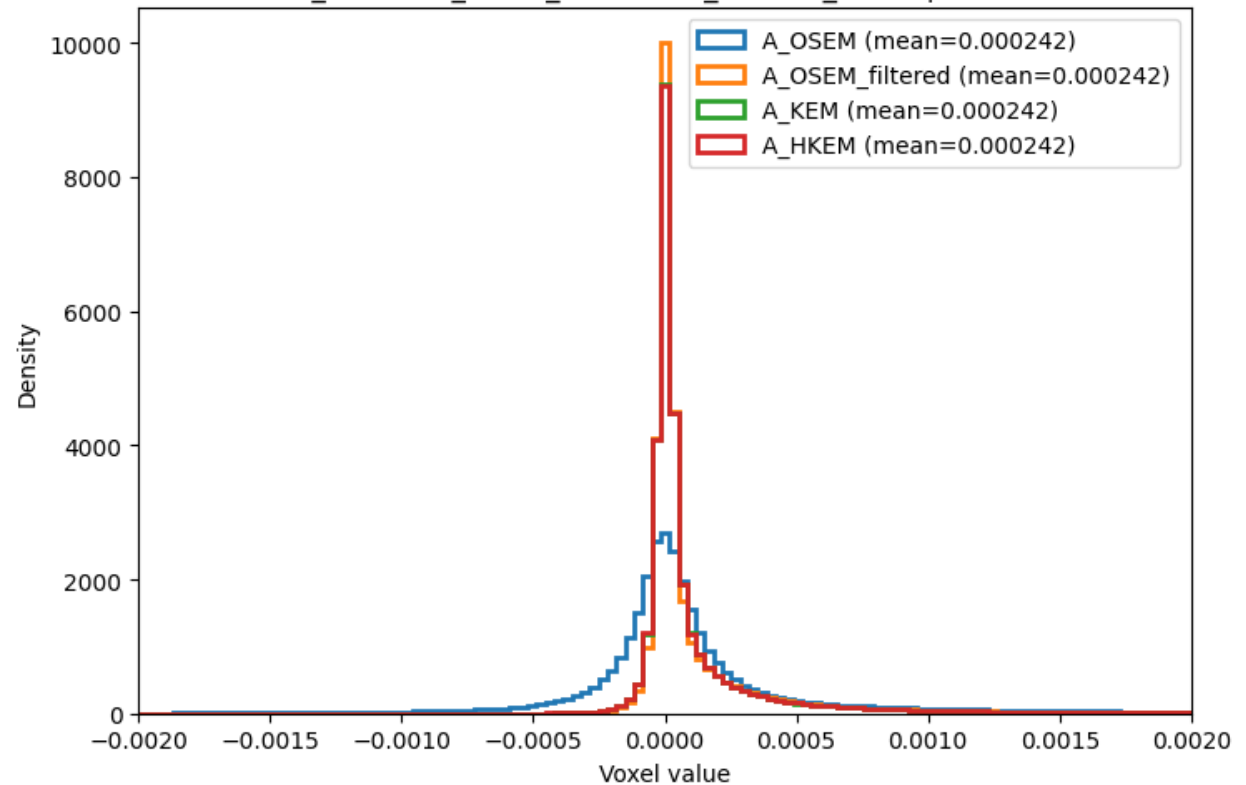
Histogram of reconstructed voxel values

- Histogram of all voxel values within the patient (5%–95% range).

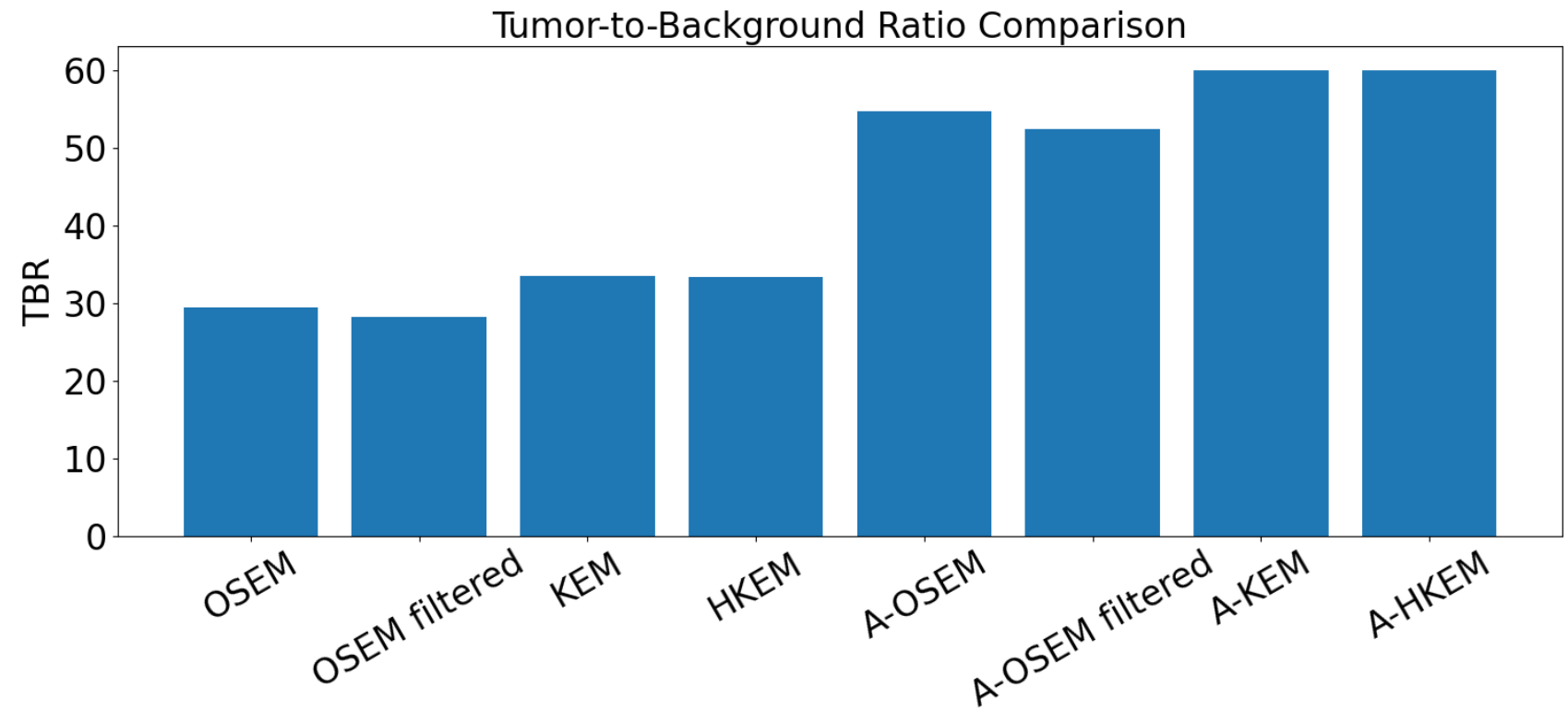
OSEM / OSEM_filtered / KEM / HKEM | Iter 20



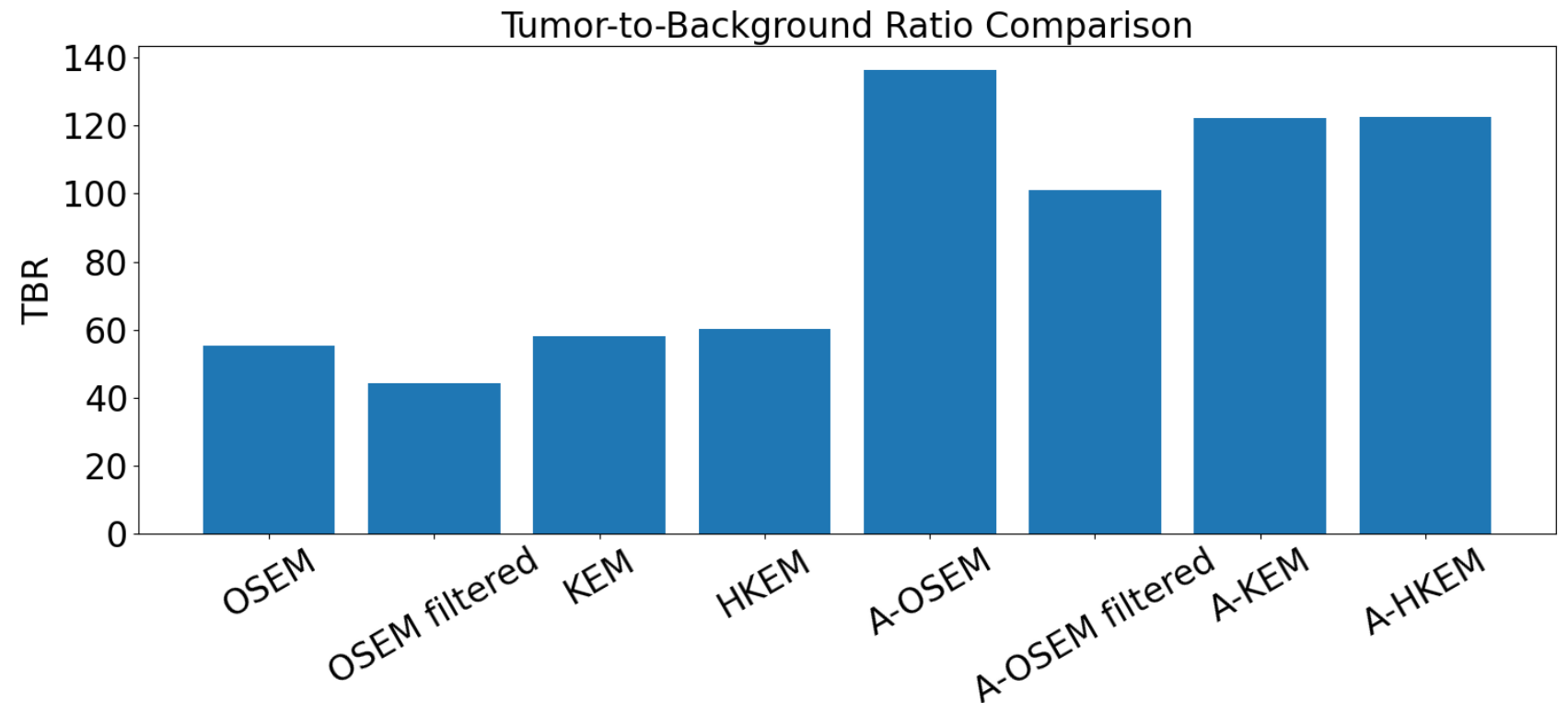
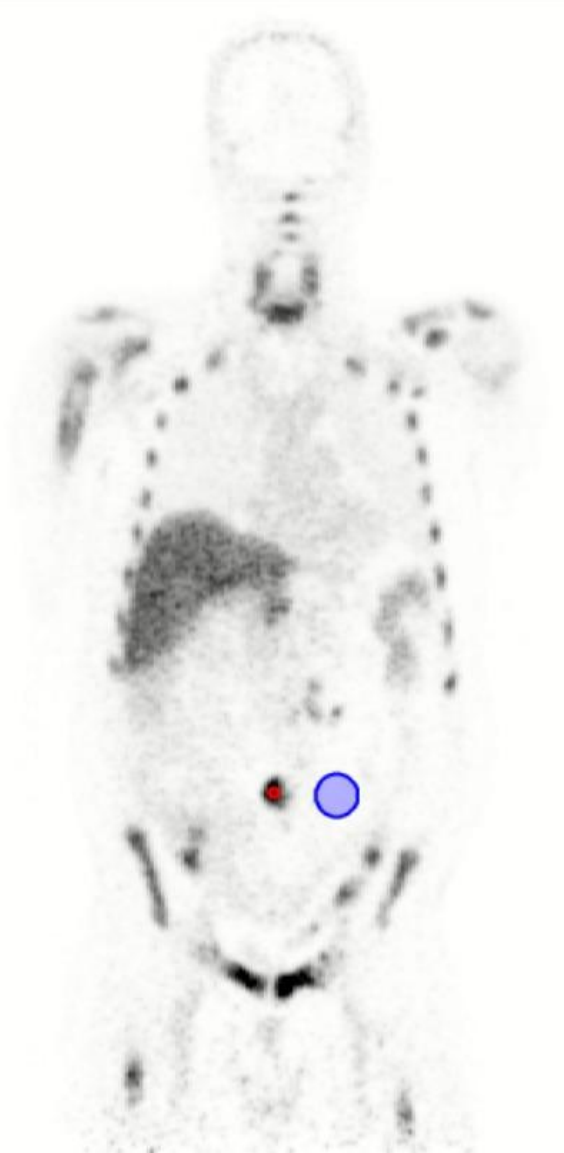
A_OSEM / A_OSEM_filtered / A_KEM / A_HKEM | Iter 20



Tumor-background ratio



Tumor-background ratio



Conclusion and future work

- Algorithms with lower bounds change the voxel value distribution by relaxing the zero lower bound.
- They produce less truncated and more symmetric voxel distributions, while kernel methods further reduces spread and improved image quality.
- Different lower bounds and kernel parameters should be investigated to achieve optimal performance.
- Different kernel designs can be explored to further evaluate and optimise performance.