Symposium on AI and Reconstruction for Biomedical Imaging, March 9-10, 2026

Bridging Physics and Learning: A Decade of Kernel Methods for Image Reconstruction

Guobao Wang, Ph.D.
Department of Radiology, UC Davis Health

March 9th, 2026 | London, UK

Ideas in Machine Learning Come in Cycles

1980–1990s

Neural Networks

1990–2000s

Kernel Methods

2010s–today

Deep Neural Networks

Where do kernel methods fit today?

Image Reconstruction: Physics-based Inverse Problem

- Imaging physics provides a forward model linking the unknown image $\boldsymbol{x}$ to measured data $\boldsymbol{y}$

$$y_i \sim \mathrm{Poisson}(\,\bar{y}_i\,), \quad \bar{\boldsymbol{y}} = \boldsymbol{P}\boldsymbol{x} + \boldsymbol{r}$$

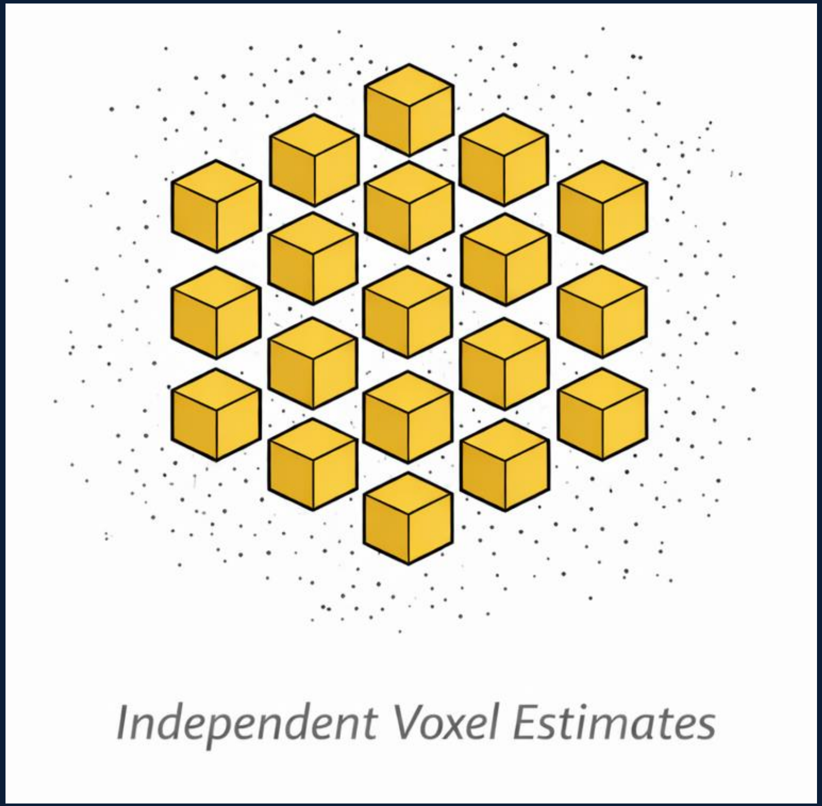
- Classic inverse problem: image reconstruction using the maximum-likelihood estimation

$$\hat{\boldsymbol{x}} = \arg\max_{\boldsymbol{x}} \, L(\,\boldsymbol{y} \mid \boldsymbol{x}\,)$$

A Geometric Perspective of Image Voxels

Maximum-Likelihood Reconstruction

- Voxels are treated independent
- Image space = Euclidean grid
- Coupling only through imaging physics



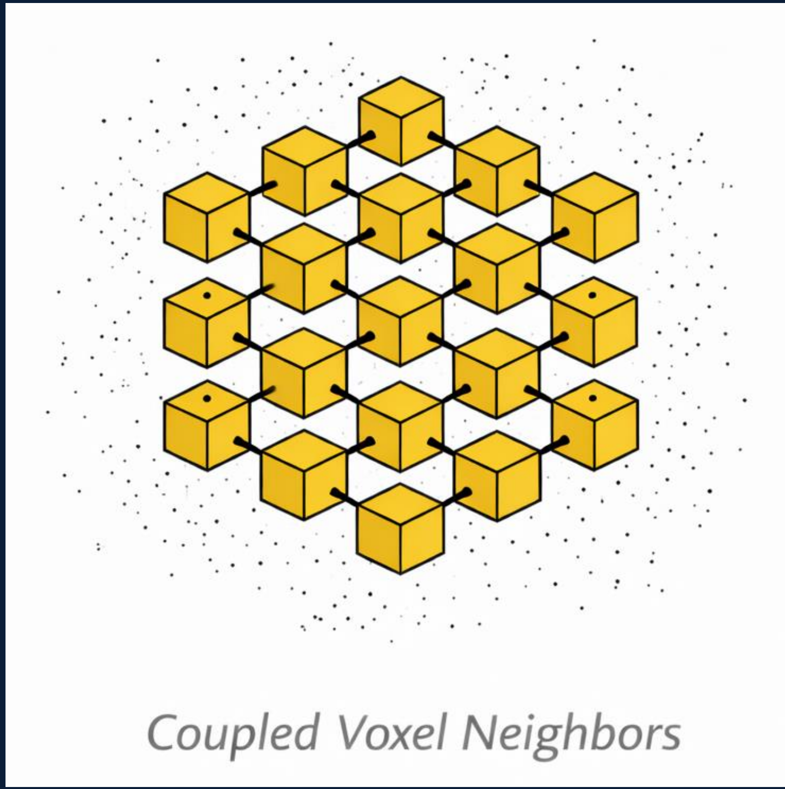
Regularization Framework

Penalized-Likelihood Reconstruction

$$\hat{\boldsymbol{x}} = \arg\max_{\boldsymbol{x}} L(\boldsymbol{y}|\boldsymbol{x}) - \beta R(\boldsymbol{x})$$

- Introducing voxel coupling via regularization
- Typically local and handcrafted
- Deep-learning prior may be used

Voxel coupling through regularization



Kernel Representation for Image Reconstruction

- Images are represented by kernels with priors built in:

$$\boldsymbol{x} = \boldsymbol{K\alpha}, \text{ where } \boldsymbol{K} \in \mathbb{R}^{N\times N}$$

- Incorporated into the physics-based forward model

$$\bar{\boldsymbol{y}} = \boldsymbol{PK\alpha} + \boldsymbol{r}$$

- Maximum-likelihood reconstruction: $\hat{\boldsymbol{\alpha}} = \arg\max_{\boldsymbol{\alpha}} L(\boldsymbol{y}|\boldsymbol{K\alpha})$

Wang GB & Qi J, ISBI 2013, IEEE-TMI 2015

Kernel Trick for Image Voxels

- Each voxel is a data sample with a feature vector $\boldsymbol{f}_j$
- The voxel intensity is linear in the transformed space:

$$x_j = \boldsymbol{w}^T \phi(\boldsymbol{f}_j)$$

- Equivalently, it is described by kernels, which computes similarity between voxels

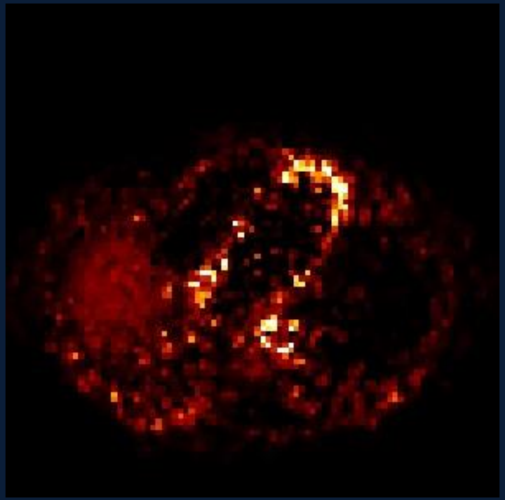
$$x_j = \sum_l \alpha_l \, \kappa(\boldsymbol{f}_j, \boldsymbol{f}_l)$$

where $\kappa(\boldsymbol{f}_j, \boldsymbol{f}_l)$ is equal to the inner product of ϕ

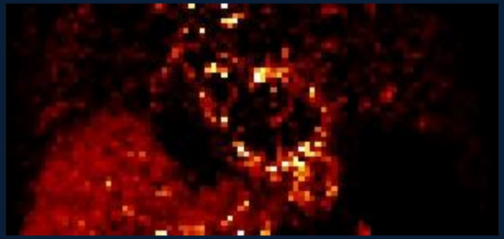
The Kernel Defines Voxel Relational Geometry

- Voxels lie in a non-Euclidean embedding space defined by image priors

3D image from a cardiac scan

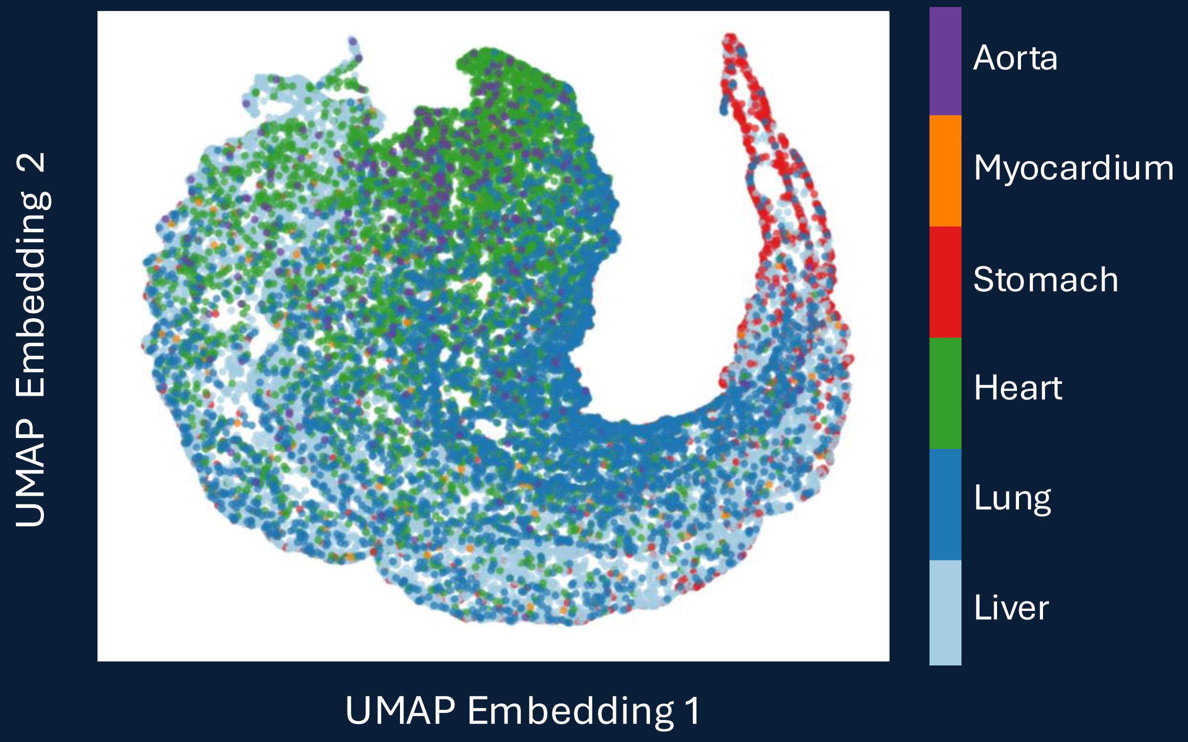


(transverse plane)



(coronal plane)

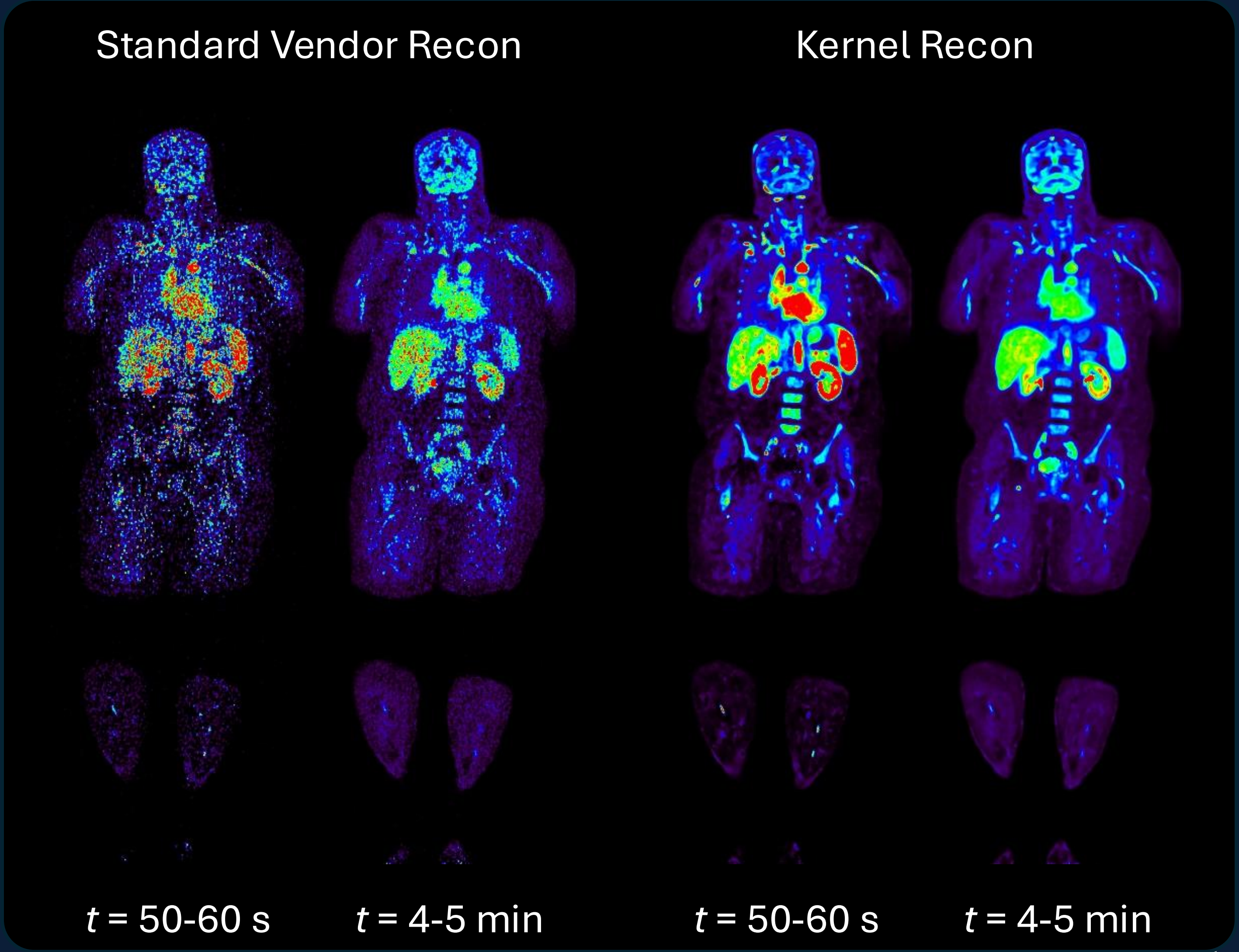
2D visualization of the voxel embedding



Application to Dynamic PET

- Image Prior:
  Composite images from longer frames
- Feature Vector:
  Temporal activity from composite longer frames
- Reconstruction:
  Dynamic PET images of shorter frames

Total-body dynamic FDG-PET Data

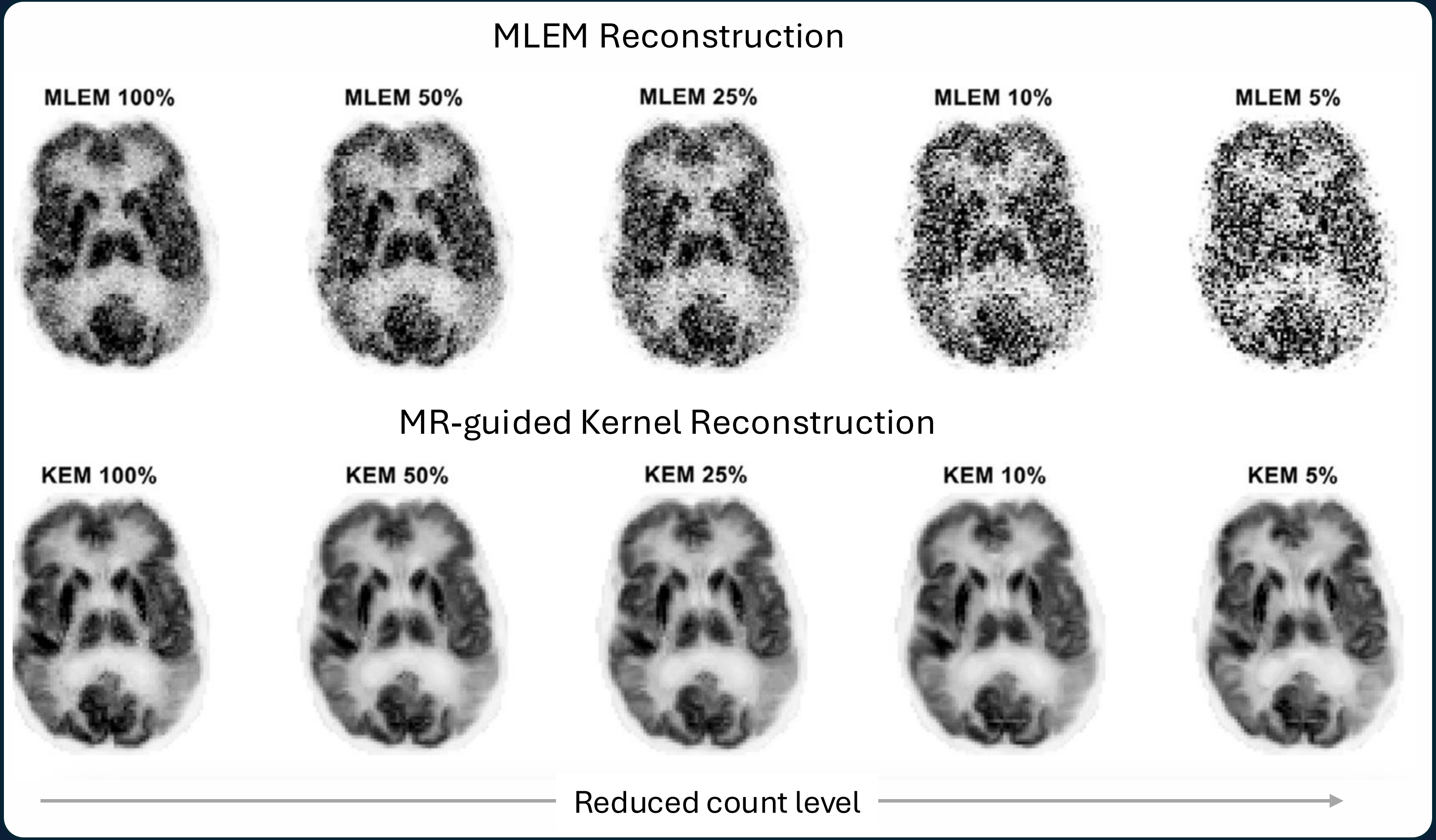


Wang & Qi 2015; Wang 2019; Zhang *et al* 2020

PET/MR

Reader & Pan, BJR 2023

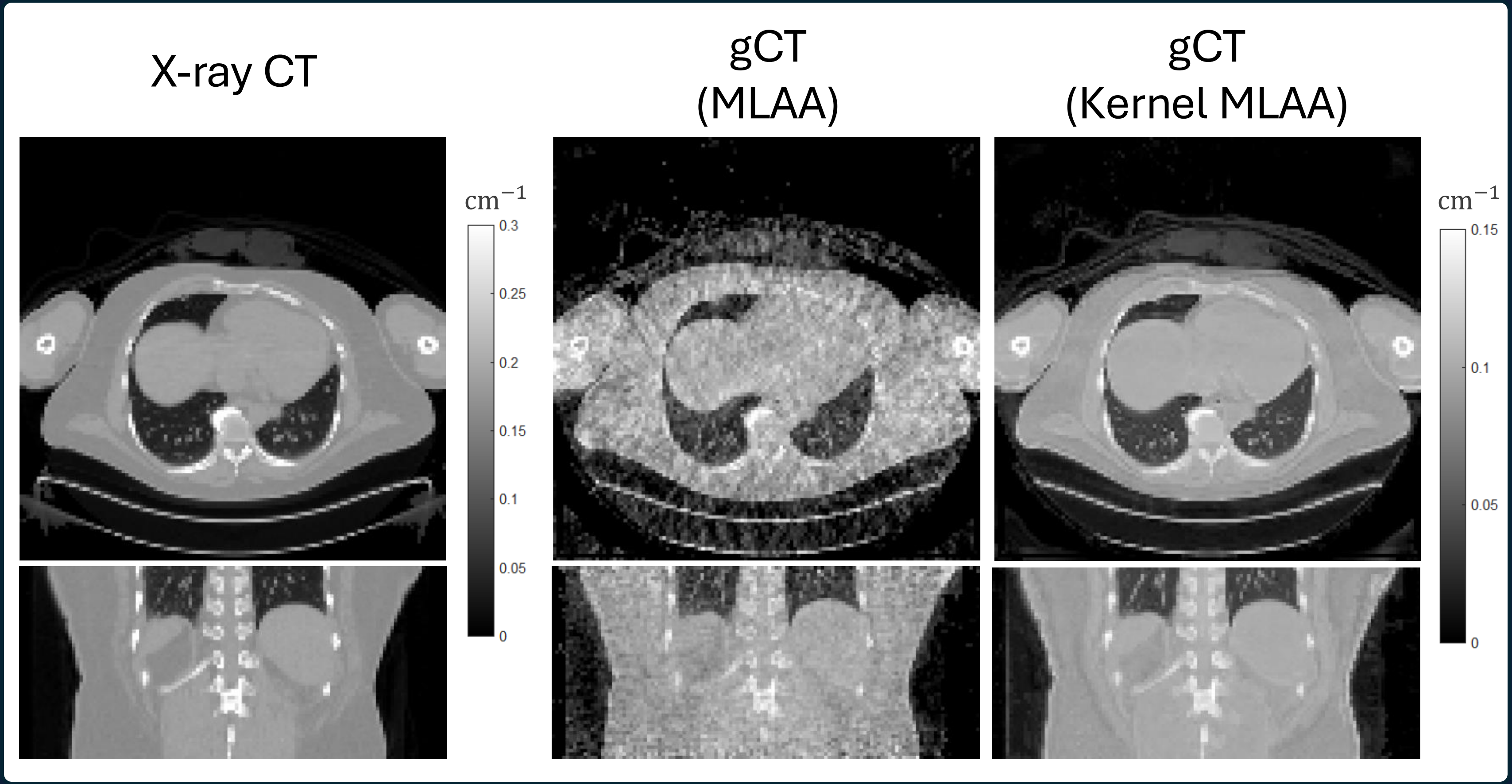
- Image Prior:
  MR images
- Feature Vector:
  Local MR patch
- Reconstruction:
  PET activity image



Hutchcroft *et al*, 2016; Novosad & Reader 2016; Bland *et al* 2017; Deidda *et al* 2018

PET-enabled Dual-Energy CT

- Image Prior:
  X-ray CT image
- Feature Vector:
  Local CT patch
- Reconstruction:
  511 keV gamma-ray attenuation map (gCT) from PET emission data



Wang GB PMB 2020; Li SQ & Wang PTRSA 2021; Li SQ *et al* IEEE-TIP 2024; Zhu Y *et al* EJNMMI 2025

Learning Voxel Representations with Deep Networks

- So far the feature vector is manually designed

$$\boldsymbol{f}_j = \text{temporal feature or MR patch}$$

- Can we learn the voxel representation?

Voxel Data from Image Prior $\boldsymbol{Z}$ → Deep Neural Network $\boldsymbol{\Phi}$ → Embedding $\boldsymbol{f}_j = \boldsymbol{\Phi}_j(\boldsymbol{Z})$ → Voxel Similarity $\kappa(\boldsymbol{f}_j, \boldsymbol{f}_l)$

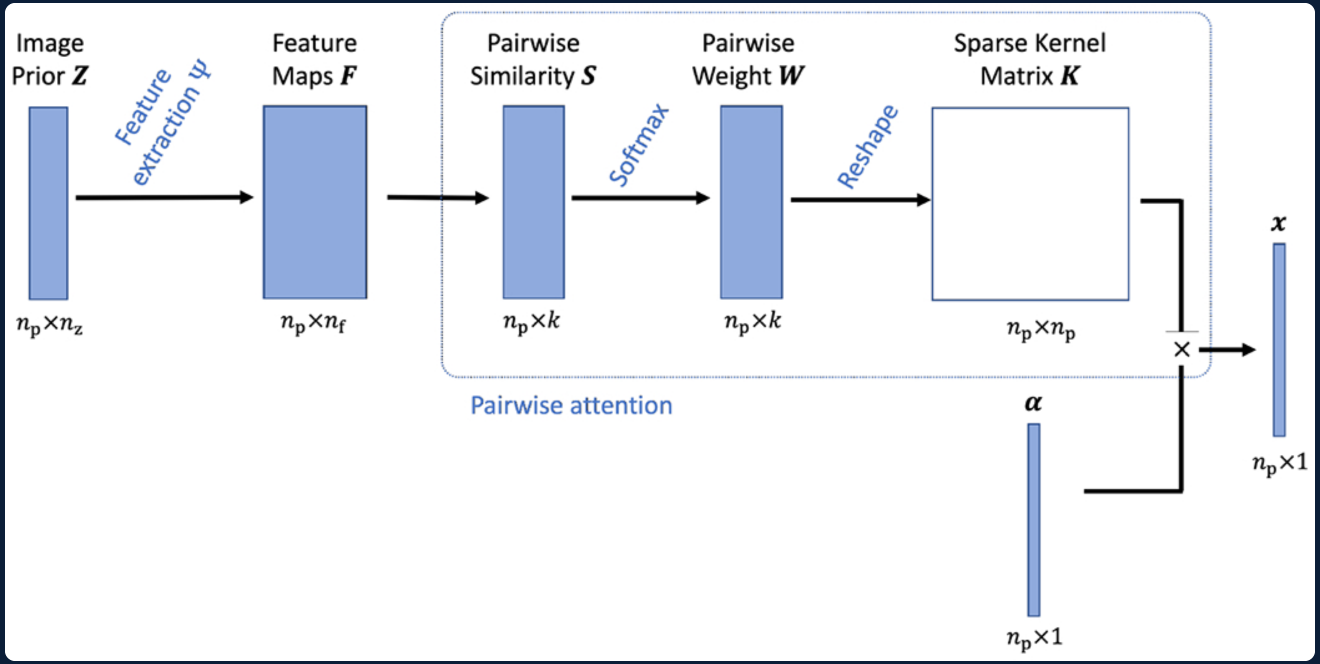
Deep Kernel Representation

- Making kernel construction trainable:

$$x = K_{\theta}(Z)\alpha$$

- Beauty: Once θ is determined, the model remains linear w.r.t. α for reconstruction

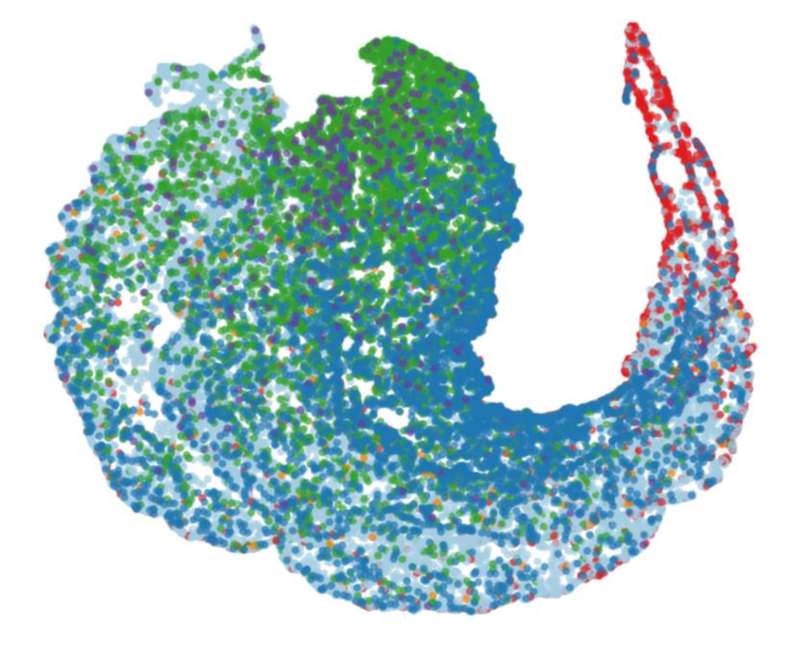
$$x = K_{\theta}(Z)\alpha$$



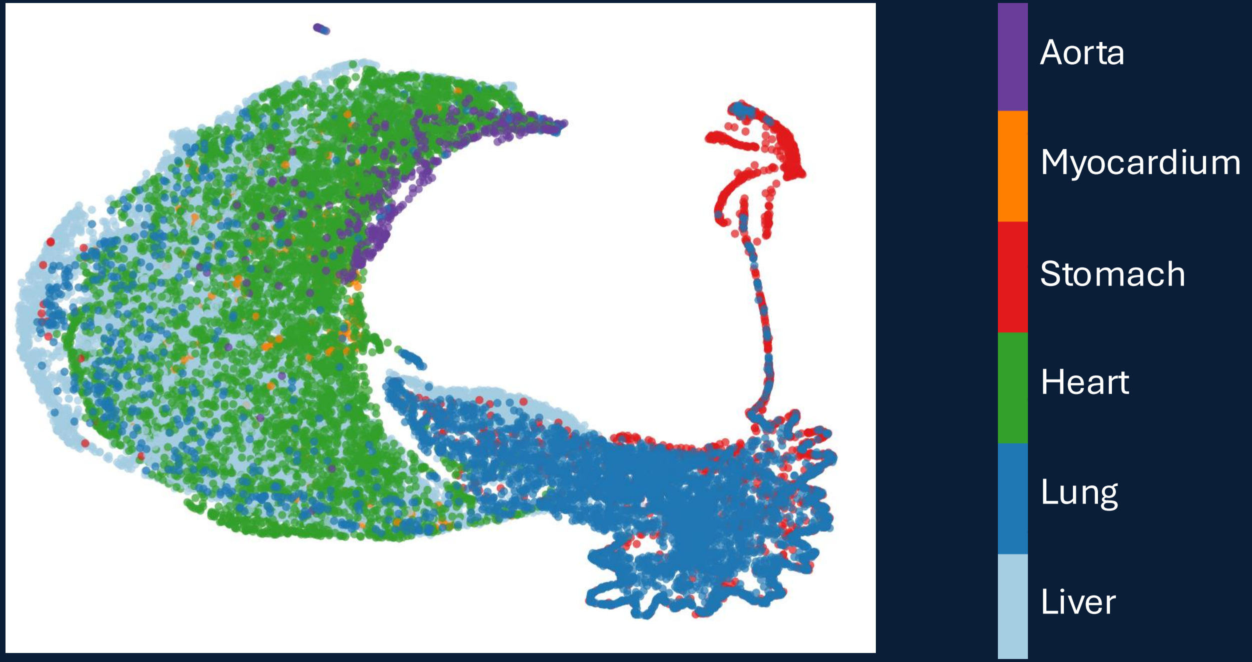
Li SQ & Wang GB, IEEE-TMI 2022

Learned Geometry of Voxel Relations

By handcrafted feature



By deep kernel learning

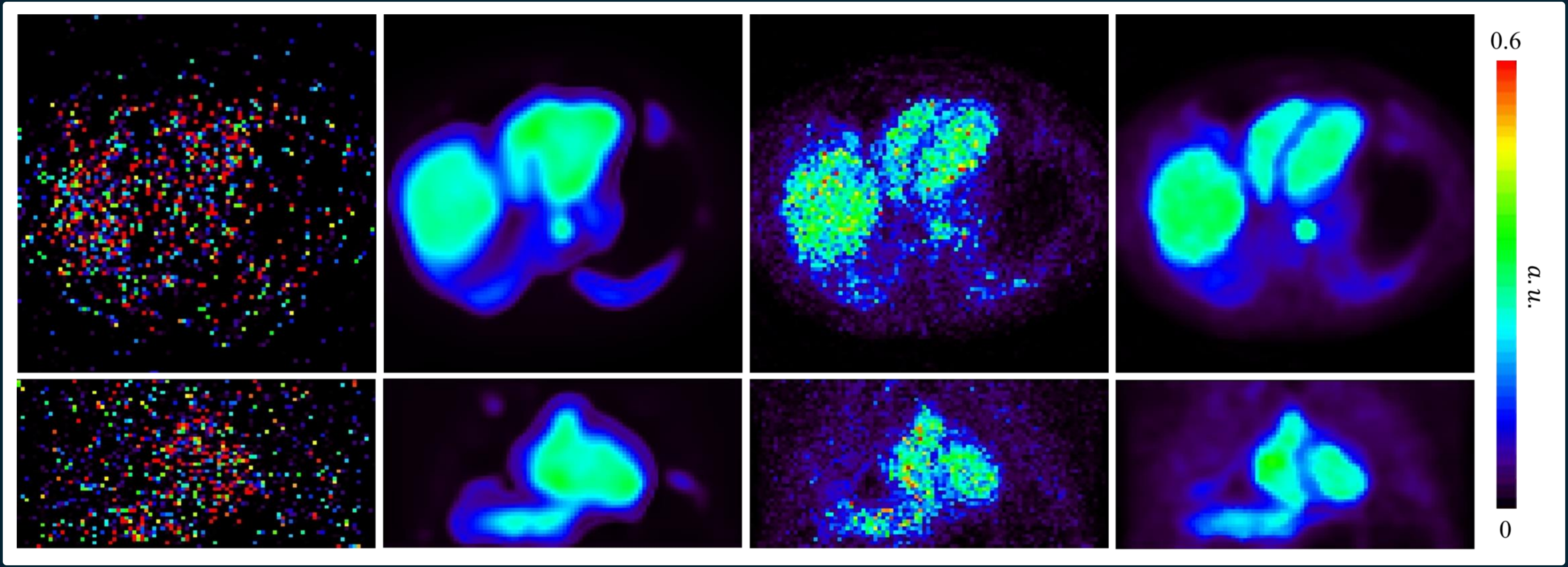


* for a cardiac FDG scan

Reconstruction Example of Patient Data by Deep Kernel

For a 2-s frame in dynamic cardiac FDG-PET imaging:

MLEM	Deep Image Prior Recon (Gong *et al* 2018)	Conventional kernel	Deep Kernel



Li SQ & Wang GB, IEEE-TMI 2022

Limitation of Single-Subject Kernel Learning

Current kernel methods rely on single-subject image prior

Challenges

- Low quality priors may lead to unreliable voxel geometry
- Online kernel learning requires additional time

Pretrained Deep Kernel Construction

Using high- and low-quality image pairs from population data (e.g., generated from total-body PET)

$$\hat{\boldsymbol{\theta}} = \arg\min_{\boldsymbol{\theta}} \sum_{n} \left\| \boldsymbol{x}_n^{\text{high}} - \boldsymbol{K}_{\boldsymbol{\theta}}\left(\boldsymbol{x}_n^{\text{low}}\right)\boldsymbol{x}_n^{\text{low}} \right\|^2$$

Two important clarifications:

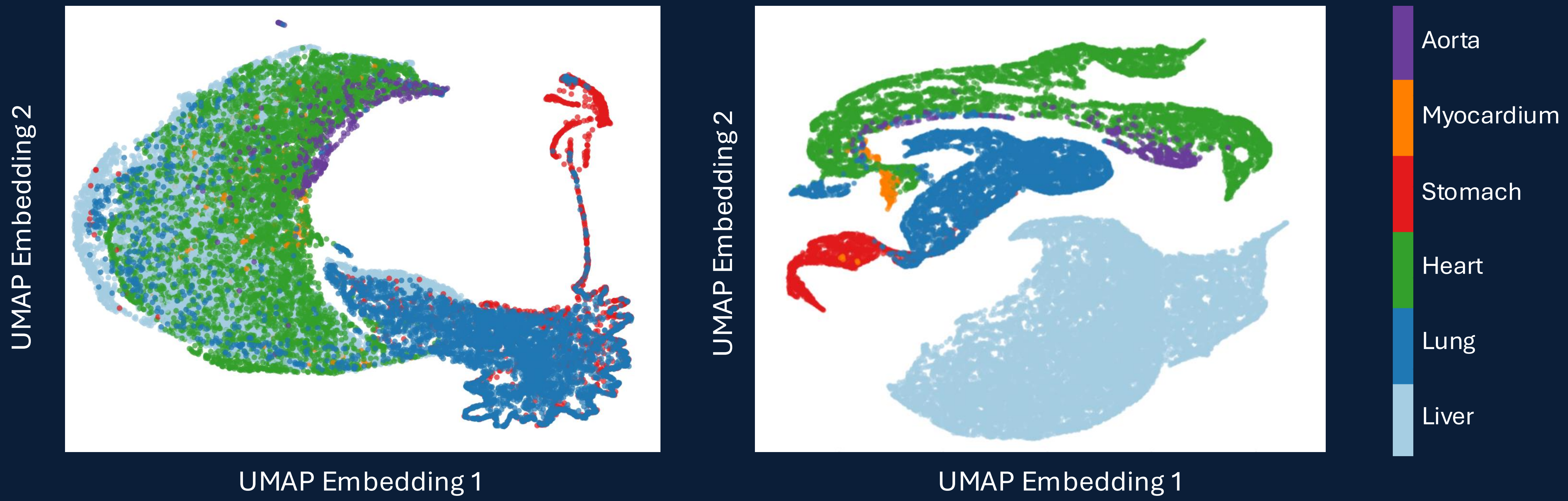
- The network learns how to construct voxel geometry, not the geometry of a particular patient
- Geometry remains subject-specific: $\boldsymbol{K}_{\boldsymbol{\theta}}\left(\boldsymbol{x}^{\text{low}}\right)$

Li SQ *et al* IEEE MIC 2024

Learned Voxel Geometry by Different Approaches

By single-subject deep kernel

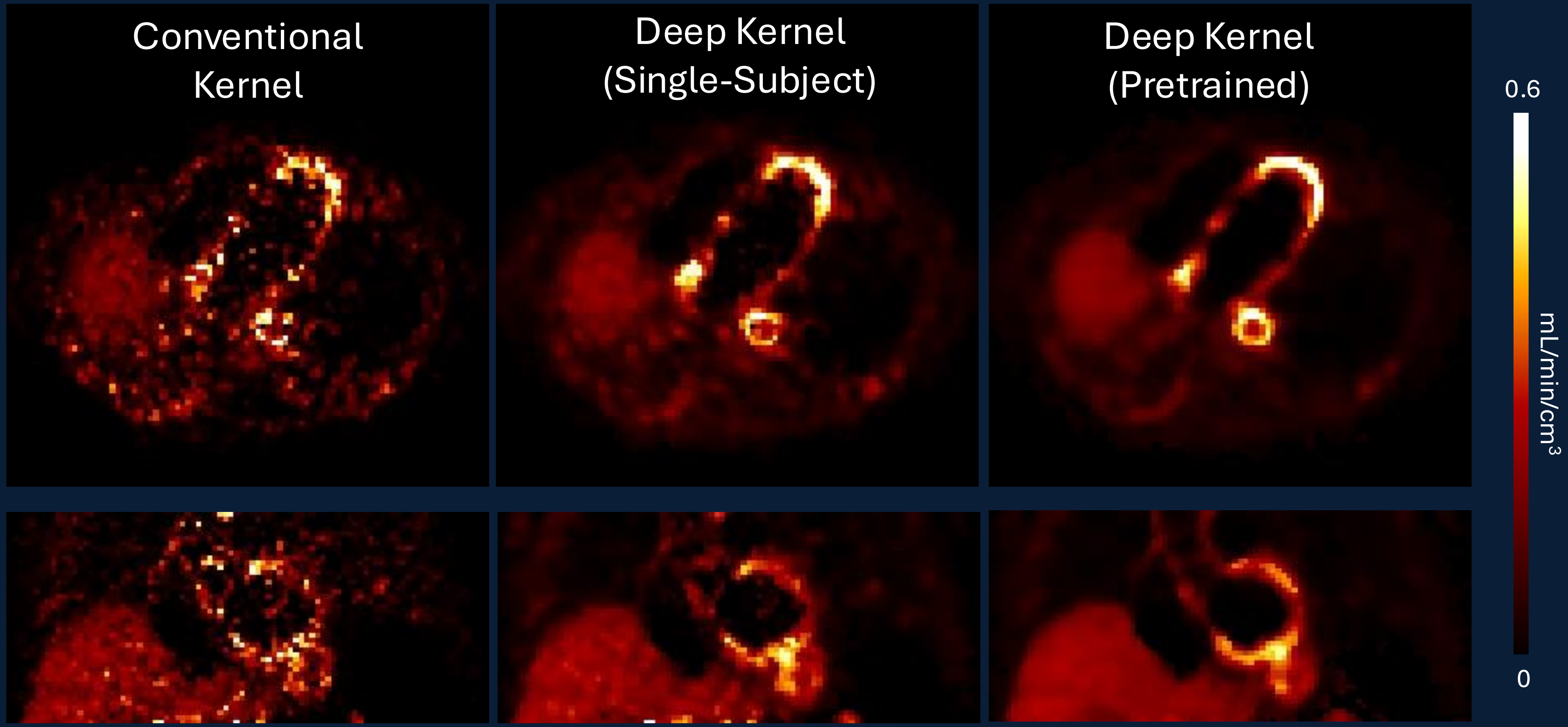
By pretrained approach



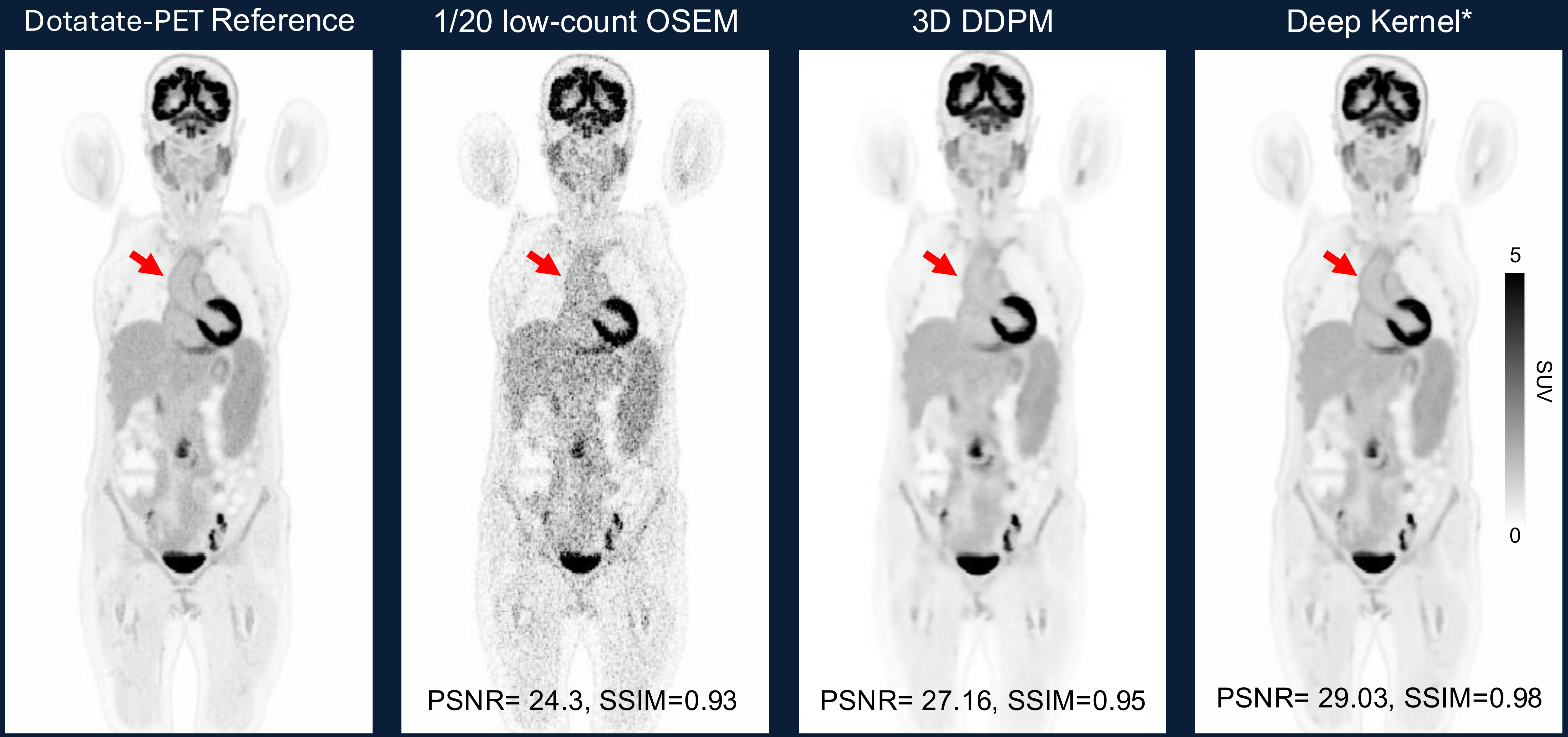
Li SQ *et al* IEEE MIC 2024

Application to Dynamic PET Imaging on Short Scanners

Parametric imaging of FDG delivery rate K_1 from dynamic images reconstructed by different approaches for a GE DST scan



Potential for Static PET Imaging



*Trained with FDG dataset. Li SQ *et al* IEEE MIC 2025

Closing Thoughts

- Many powerful deep denoisers have been trained for image denoising
- Common use in reconstruction
  - ➢ Plug-and-Play
  - ➢ Denoising by Regularization (RED)

Can we extract voxel relational geometry from deep denoisers?

Thought Experiment

- Taylor expansion of a deep denoiser

$$\boldsymbol{D}_{\boldsymbol{\theta}}(\boldsymbol{x}) \approx \boldsymbol{D}_{\boldsymbol{\theta}}(\boldsymbol{x}_0) + \boldsymbol{J}(\boldsymbol{x}_0)(\boldsymbol{x} - \boldsymbol{x}_0)$$

where $\boldsymbol{J}(\cdot)$ is the Jacobian

- Inspired by the neural tangent kernel (Jacot *et al.*, 2018),

$$\boldsymbol{K} = \boldsymbol{J}(\boldsymbol{x}_0)\boldsymbol{J}(\boldsymbol{x}_0)^T$$

may encode voxel relational geometry

Summary

Kernel Reconstruction

= Imaging Physics × Voxel Relational Geometry

Acknowledgment

Current or former members in Wang Lab who contributed:

Dr. **Siqi Li** (Current)
Dr. Yansong Zhu (Current)
Dr. Benjamin A. Spencer

Siqi Li, PhD

Collaborators / Senior Advisors

UC Davis
Dr. Ramsey D. Badawi
Dr. Simon R. Cherry
Mr. Will Hutchcroft
Dr. Gong Kuang (now UF)
Dr. **Jinyi Qi**

UIH
Dr. Yang Lv
Dr. Xiaoyun Long

UCSC
Shiva Abbaszadeh

Jinyi Qi, PhD

Funding Support (in part):

NIH R01DK124803
NIH R01EB033435
NIH R01EB036562
NIH R21EB027346
NIH R21HL131385
AHA BGIA25780046