Training School @ 10-12-2025
Positron Emission Tomography and Single Photon Computed Tomography

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on behalf of the
Collaborative Computational Projects on
Synergistic Reconstruction for Biomedical Imaging (CCP SyneRBI)

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http://www.ccpsynerbi.ac.uk

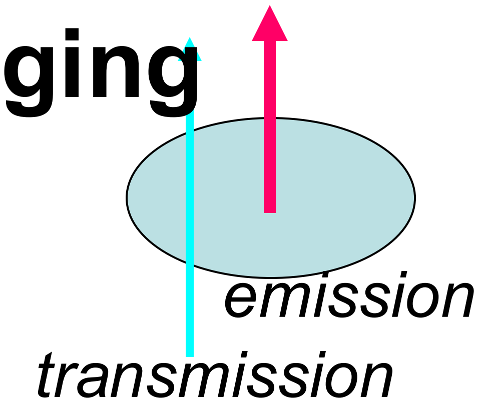
Functional Tomographic Modalities

Label ‘foreign’ substance and “see” where it is in the subject.

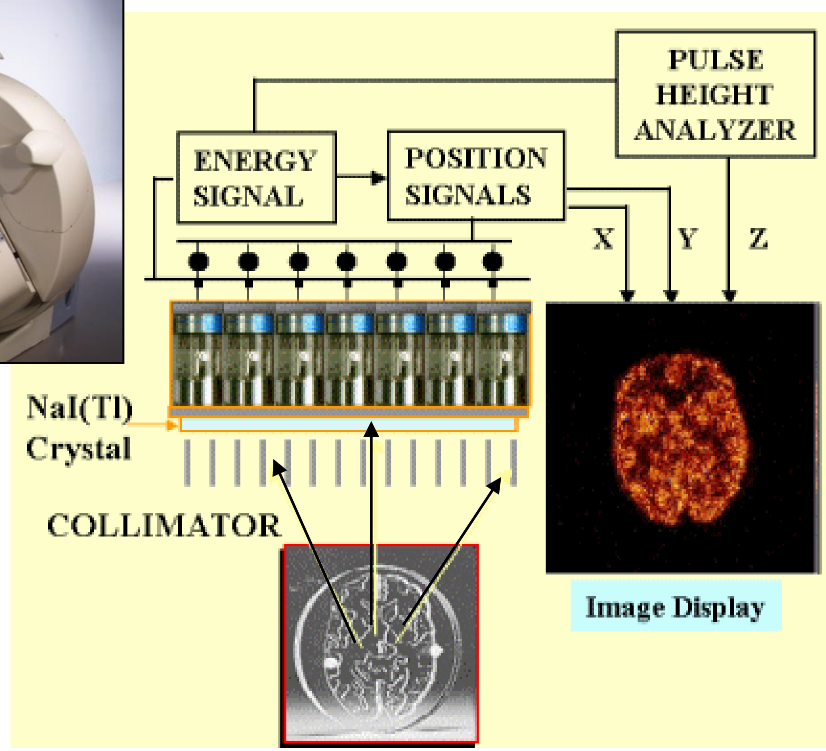
Molecule design allows you to see different functional pathways.

- SPECT (Single Photon Emission (Computed) Tomography)
  - Substance labelled with radio-nuclide (photon emission)
  - Relatively cheap but relatively low resolution and sensitivity
- PET (Positron Emission Tomography)
  - Substance labelled with radio-nuclide (positron emission)
  - Expensive due to scanner and cyclotron
- MRI Spectroscopy
- Functional MRI

Nuclear Medicine: planar imaging



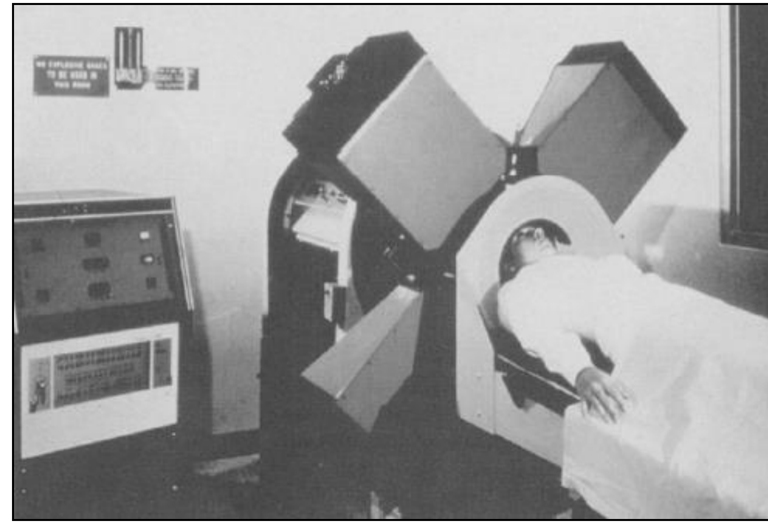
- radioactive tracer administered to patient
- scintillation detector detects gamma ray emission
- image displays *in-vivo* distribution of activity
- tracer distribution and variation with time represents organ function or active molecular process

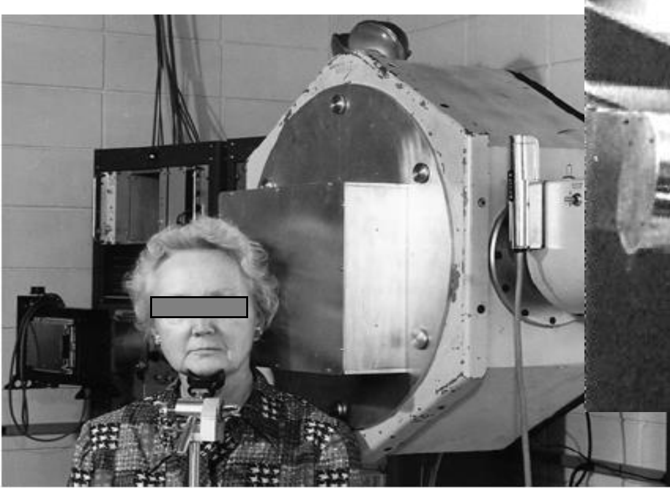


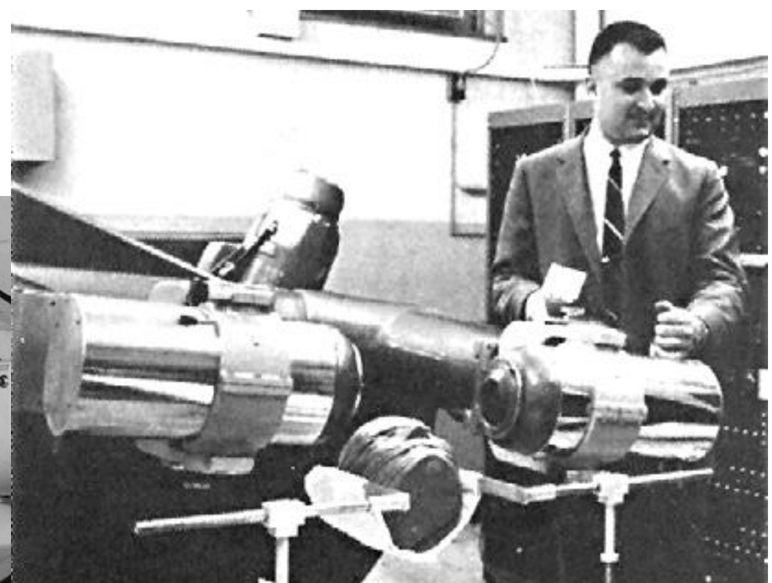
Tomography

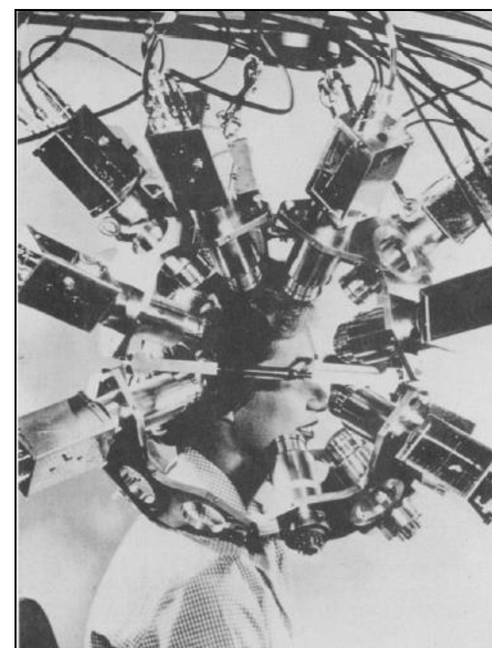
Form a representation of slices through the subject

- Anger camera 1958
- Positron counting, Brownell 1966
- Tomo reconstruction; Kuhl & Edwards 1968
- First rotating SPECT camera 1976
- PET: Ter-Pogossian, Phelps 1975



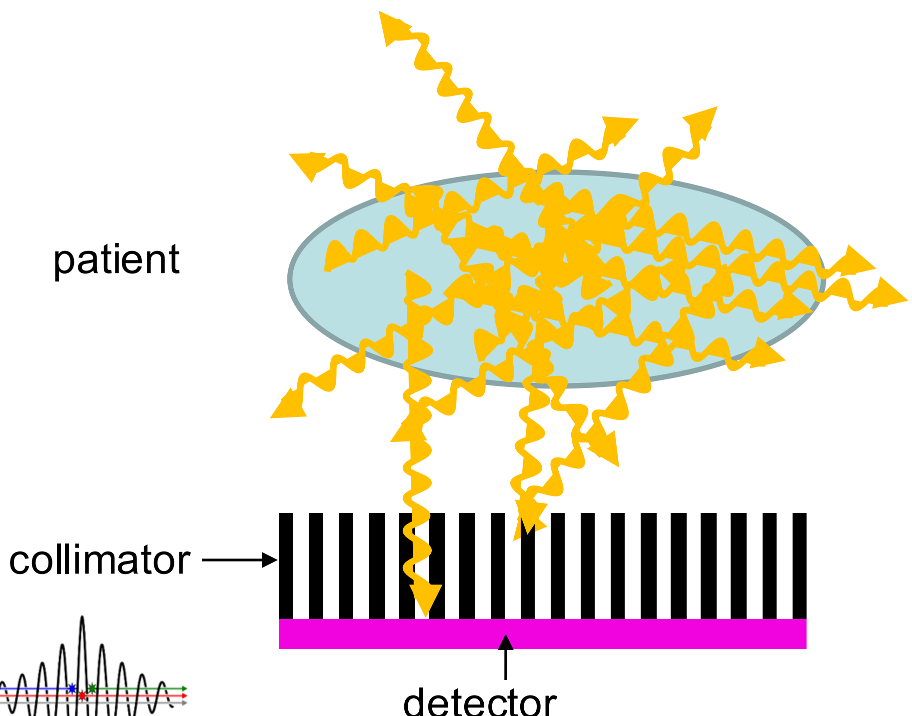






Single Photon Emission Computed Tomography (SPECT)

- Substance labelled with radio-nuclide (*single photon emitter*)
- Radio-active decay is a random (*aka* stochastic) process



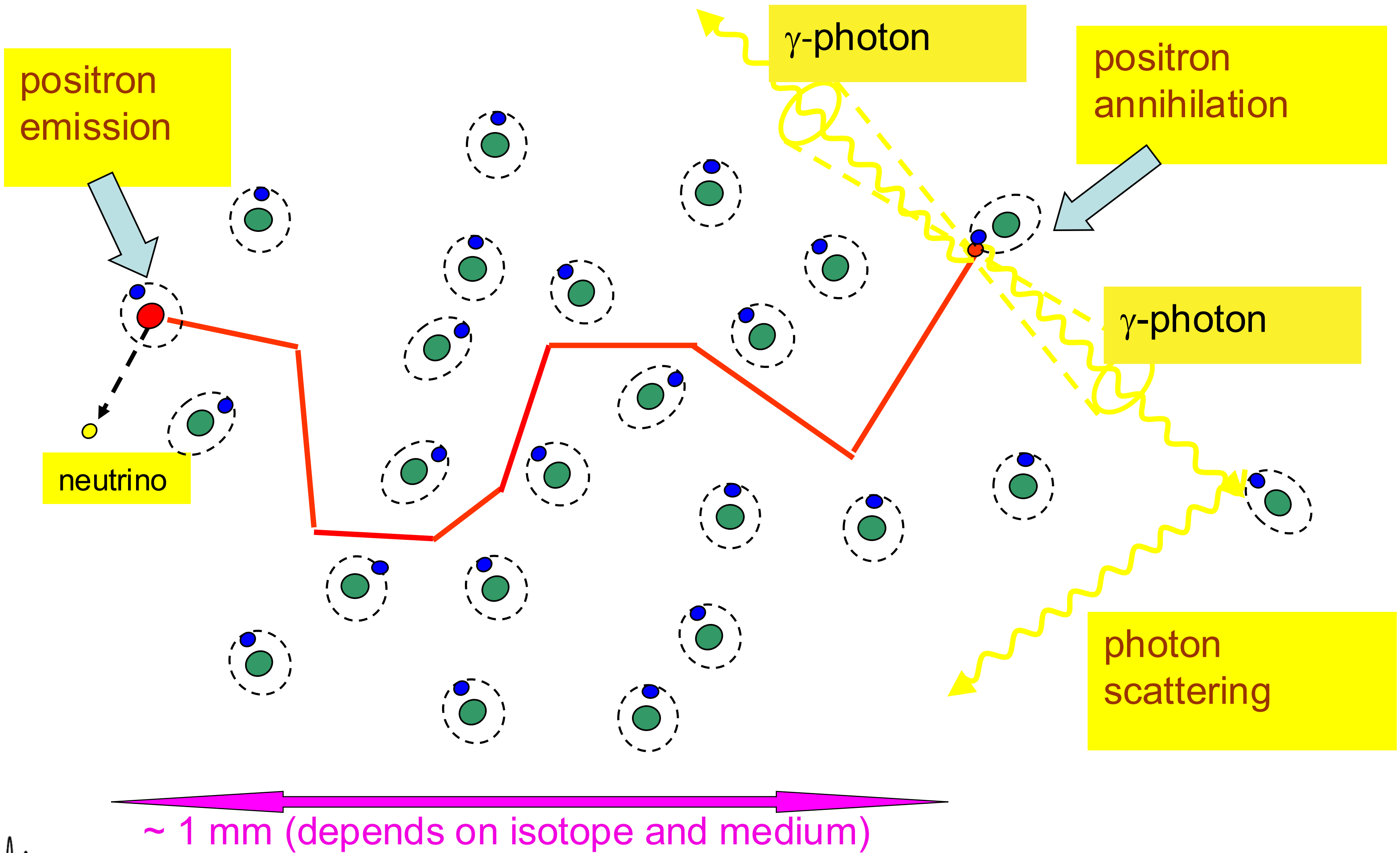
Single Photon Emission Computed Tomography (SPECT)

- – Counting photon by photon
- – Need a collimator to get spatial info
  - Resolution/sensitivity trade-off
  - Clinical spatial resolution of ~10mm
    (pinhole collimators can give ~1mm resolution)
- – Acquisition duration 5min - 30min
- – Radio-nuclides: Technetium-99m, Thallium-201 , …
  Medium half-life (6 hours or more)
- – Example clinical applications:
  - cardiac perfusion (MyoView, Sestamibi)
  - Parkinson’s (DATScan)
  - Blood flow (Ceretec)

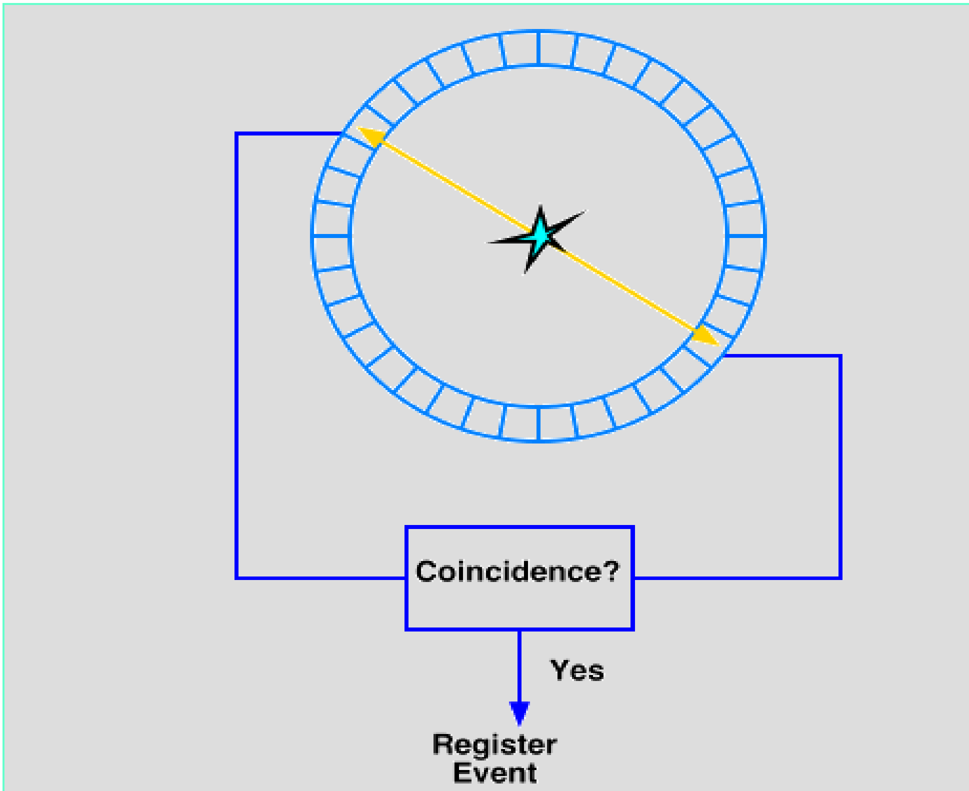
Positron Emission Tomography (PET)

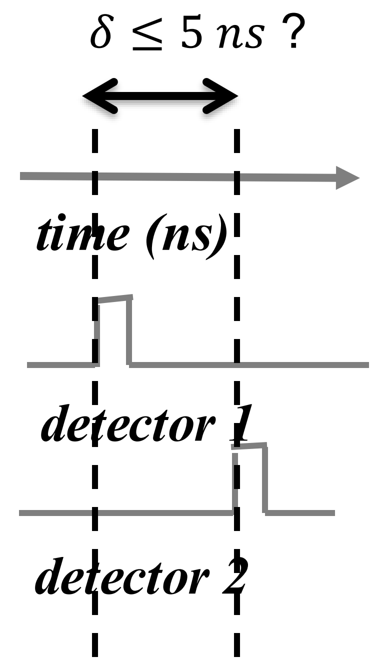
- Substance labelled with radio-nuclide (*positron emission*)
- Radio-active decay is a random process
- Detection of photon-pair, no collimator necessary
- Radio-nuclides: C-11, O-15, F-18, Ga-68…
- Spatial resolution of ~5mm
- Temporal resolution 5sec-2min
- Example clinical applications:
  - Oncology, e.g. cancer staging)
  - Neurology, e.g. dementia)
  - Cardiology, e.g. myocardial viability (infarct size) or perfusion

PET physics in a nutshell



Coincidence Detection





“prompt” coincidence event if two gammas detected within short time (~5 ns)

	SPECT	PET
Mechanism	Single photon	Positron (2 collinear photons after annihilation)
Photon energy	Isotope-dependent (140-250 keV)	511 keV ($E = mc^2$)
Radio-isotopes	Tc-99m, Tl-201, …	F-18, C-11, Rb-82 …
Collimation	Physical	Electronic
Resolution/noise	☹	☺
Cost	☺	☹

Further reading: Rahmim & Zaidi, PET versus SPECT: strengths, limitations and challenges, Nuclear Medicine Communications 2008, 29:193–207

Data storage

- Histogrammed
  - counts detected in a certain time frame
  - often called “sinograms” or “projections”
    - 4D: TOF, (2D) sinogram, view, radial
    - sinogram-index runs over “axial positions” and “ring-differences” (*aka* segments)
- List mode data
  - A list of all coincidences
  - Currently no listmode reconstruction in SIRF but coming soon!
- Attenuation image
- “Normalisation” (or “calibration”) files

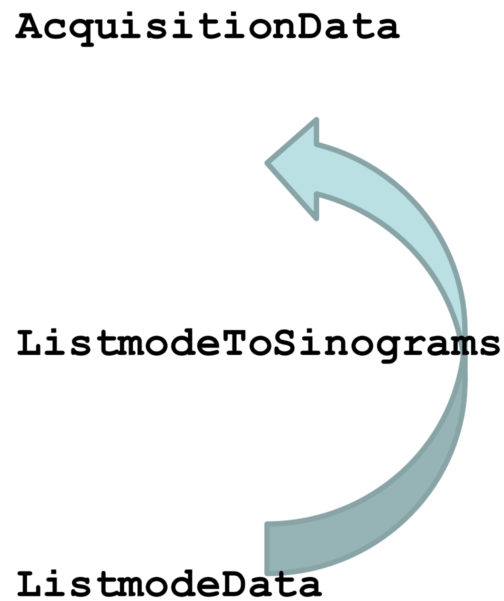


Image reconstruction algorithms

- Aim:

  Construct ‘image’ which estimates radioactivity distribution (kBq/ml) in the subject

- Two different classes
  - ‘Analytic’, e.g. Filtered Back Projection (FBP)

    Based on geometrical inversion formulas

    Fast, linear, but low quality and inflexible
  - ‘Statistical’ or ‘Iterative’, e.g. Maximum Likelihood

    Based on statistical estimation theory

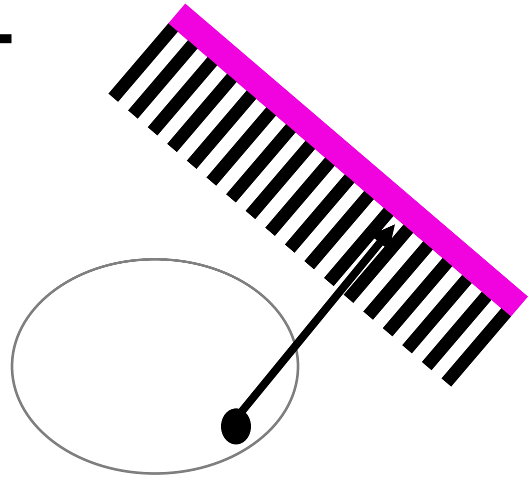
    Use ‘measurement model’ and how to treat ‘noise’, and maybe other information (e.g. anatomical)

    Try to find ‘most likely’ image by repeated adjustments

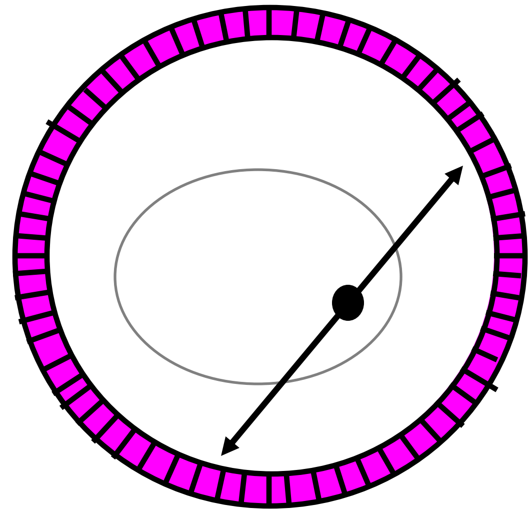
    Slow, non-linear, but potentially higher quality and flexible

What are the data? projections

SPECT



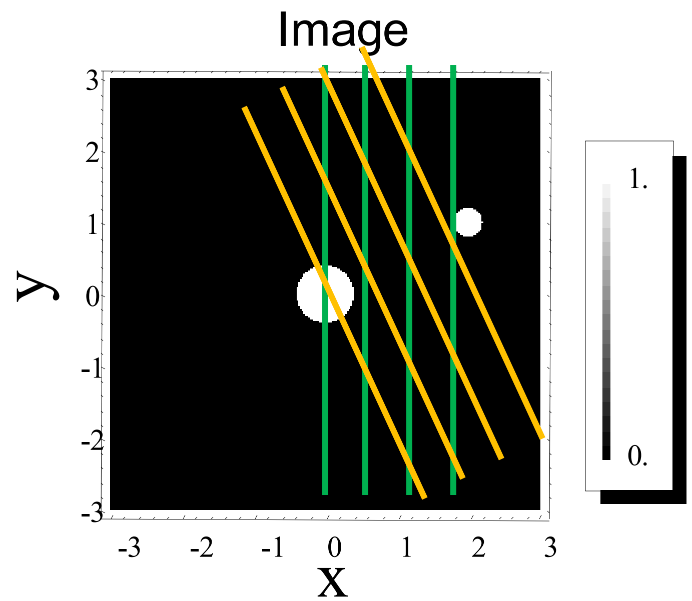
PET

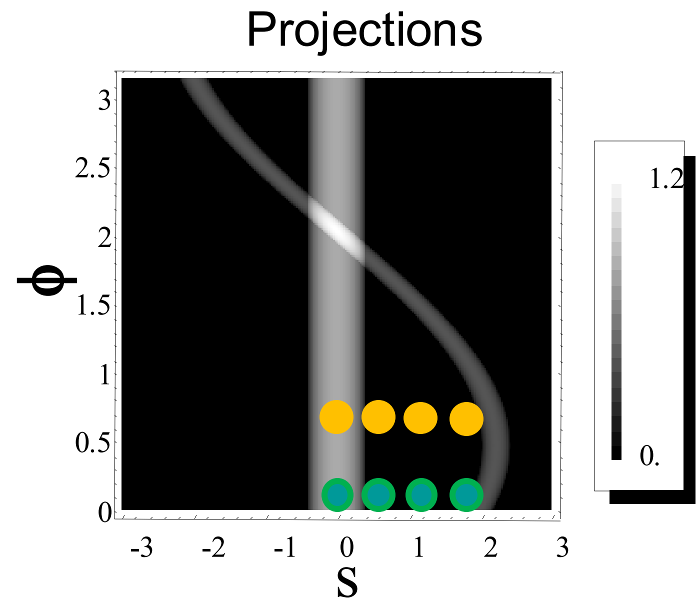


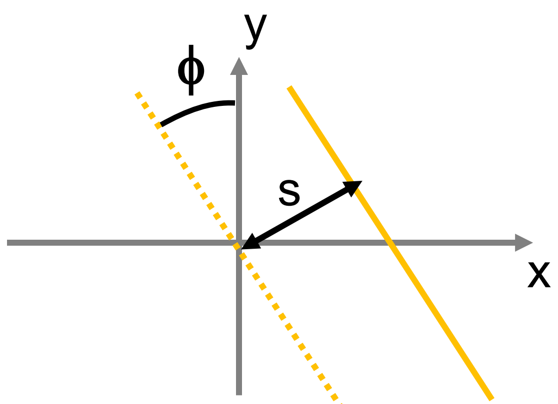
- For each detected event, the emission occurred on a ***Line Of Response*** (LOR)
- The mean number of detected events will therefore be proportional to the accumulated activity on that LOR
- Hence, the (mean of the) data are (proportional to) ***line-integrals through the activity image***.

Projection data (sinogram)

Line integrals through the object (X-ray transform)

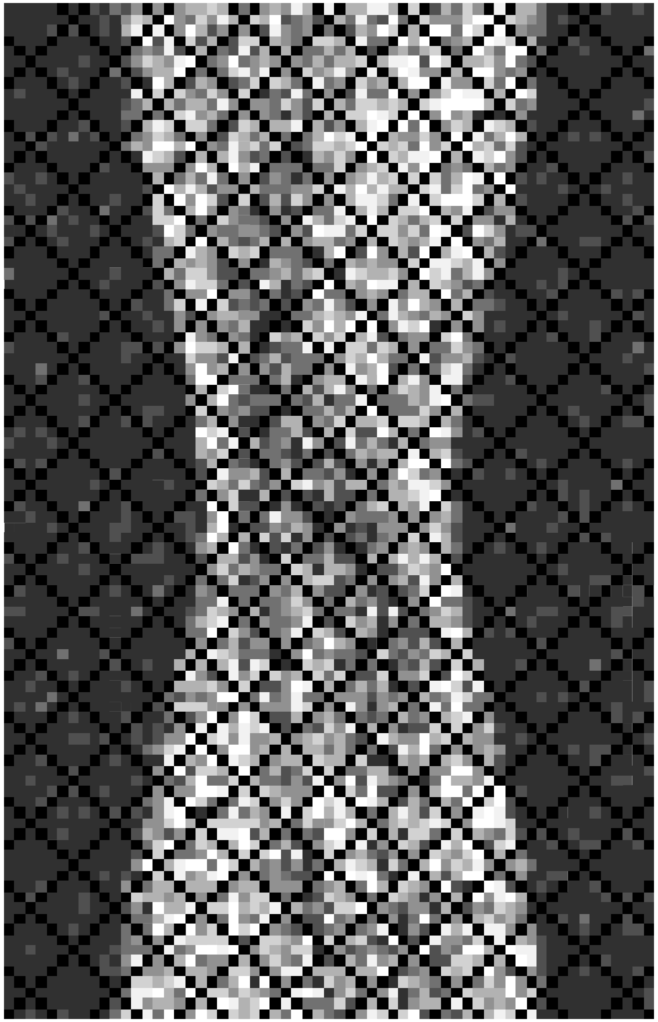






What are the data?

- Projections
  (basis for derivation of FBP)
- Add Attenuation, scatter, randoms (PET)
- Detection efficiencies
  (e.g. defective detector block)
- Gaps between blocks (PET)
- Noise!



SPECT or PET acquired data model

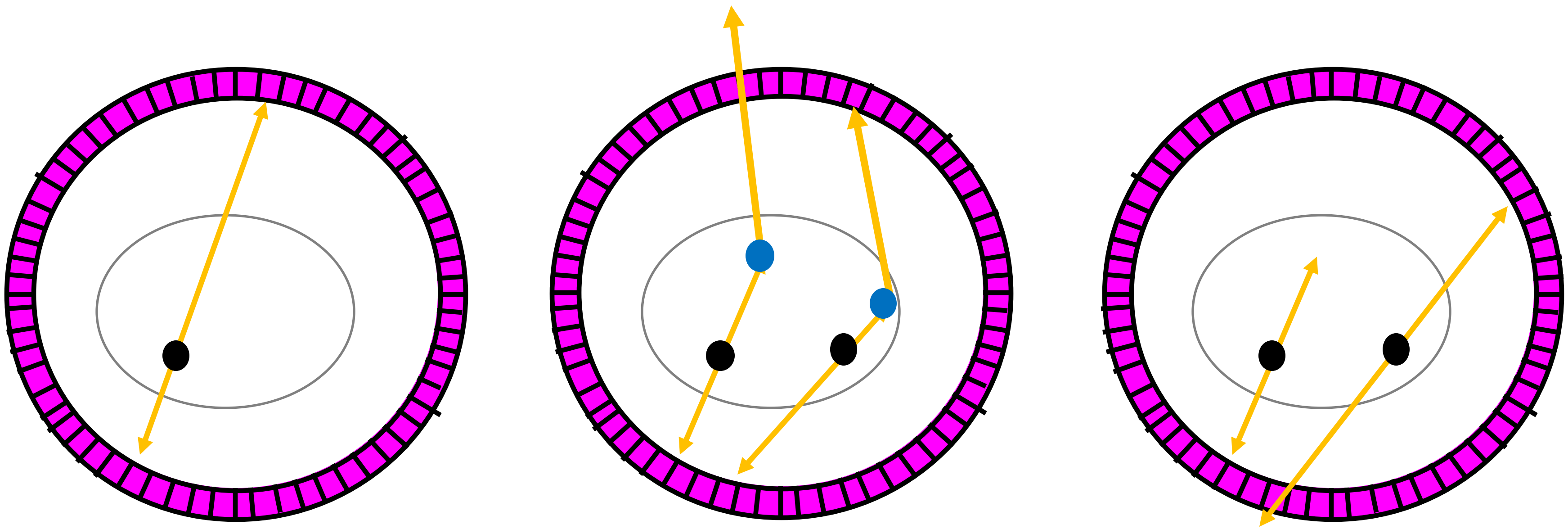
- Measured data y (**`AcquisitionData`**) is Poisson distributed
- Given an image x (**`ImageData`**), the **`AcquisitionModel`** can be used to compute the "mean" of the data

$$\bar{y} = A\,x + b$$

`AcquisitionModel.forward(x)`

  - A : "line integrals", attenuation, detection efficiencies, resolution modelling
  - b : mean "randoms" and "scatter"

Types of PET coincidences



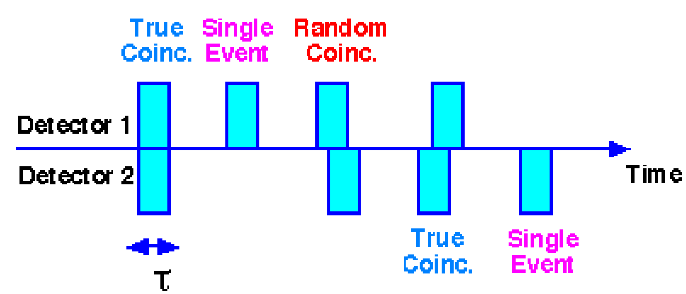
True (unscattered)

Scattered

Accidental
or “random”

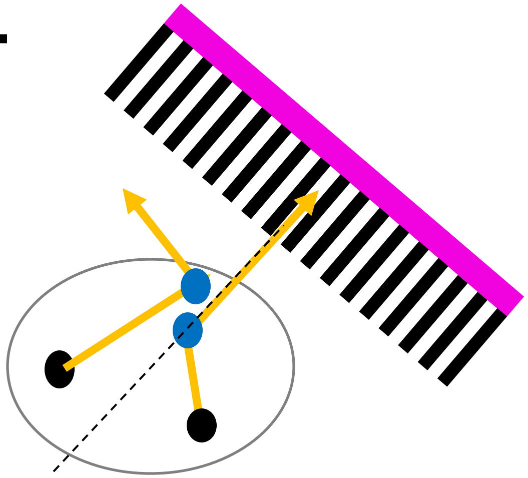
Measured data:

“Prompts” = Trues + Scatters + Randoms

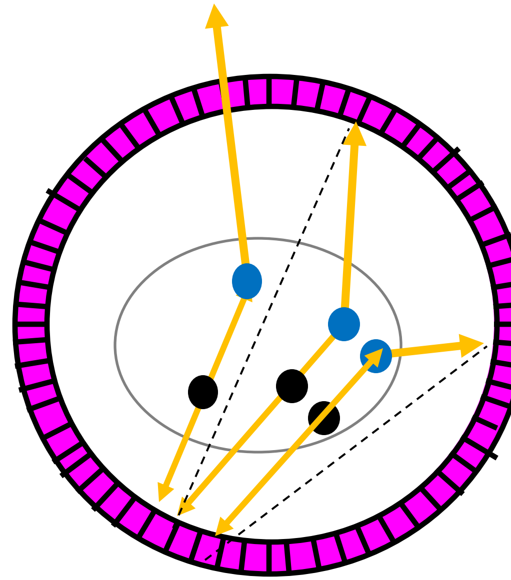


Compton scatter

SPECT



PET

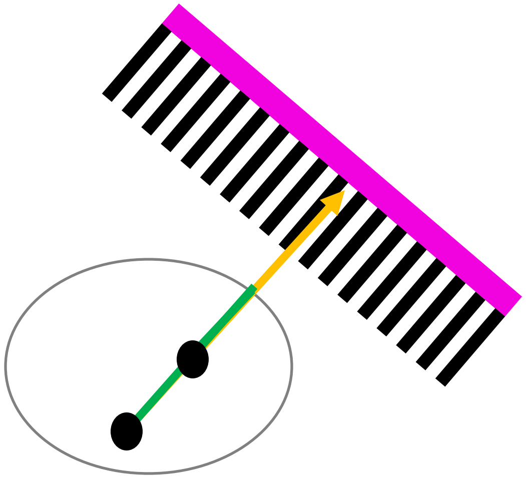


2 possible cases for scattered photon:

- Not detected
  - *Most likely (~80-90% of the time)*
  - *Effect: detected counts are too low*

- Detected within the energy window
  - *Less likely*
  - *Effect: scattered detected counts give wrong spatial information*

Attenuation modelling in SPECT



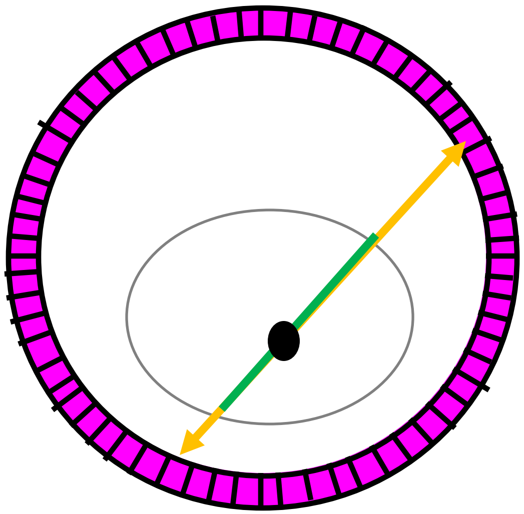
Different points on a LOR have different attenuation

$$\bar{y} \sim \sum_{v \text{ on LOR}} a_v \, \lambda_v + \bar{s}$$

λ_v is the activity in voxel v

The measured data y can therefore **NOT be precorrected** for attenuation. Problem for FBP!
(No problem for MLEM/OSEM/etc)

Attenuation modelling in PET



Photon 1: $a_1 = \exp(-\int_r^{r_{det1}} \mu(l)dl)$

Photon 2: $a_2 = \exp(-\int_r^{r_{det2}} \mu(l)dl)$

Both photons

$$a = a_1 a_2 = \exp\left(-\int_r^{r_{det1}} \mu(l)dl - \int_r^{r_{det2}} \mu(l)dl\right)$$

$$\mathrm{a} = \exp\left(-\int_{r_{det1}}^{r_{det2}} \mu(l)dl\right)$$

All points on a LOR have ***the same*** attenuation

$$\bar{y} \approx a \sum_{v \text{ on LOR}} \lambda_v + \bar{s}$$

The measured data ***can*** therefore be precorrected for attenuation. Better for FBP!

(Still no problem for MLEM/OSEM/etc)

Iterative image reconstruction

Basic idea:

- We can ***model*** the acquisition process:

  "*given an image, this is what I will measure*"

  estimated_data =

  forward_project(image)+background

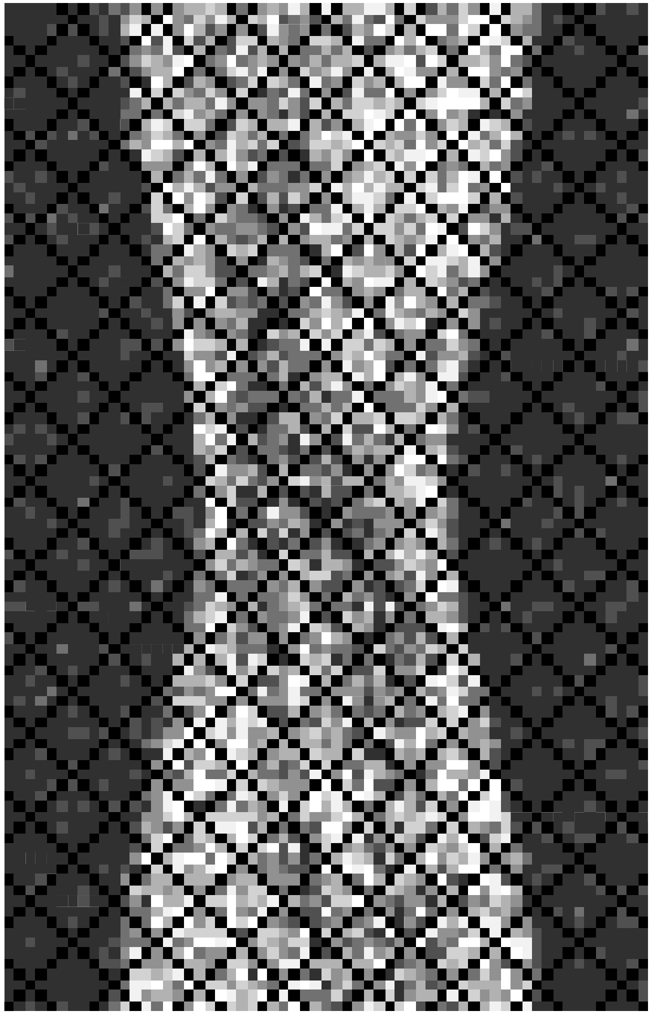
- Try to find an *image* such that we ***fit*** the *measured data*:

  measured_data ~ estimated_data

This fit has to be done iteratively.

What is a good fit?

- “goodness-of-fit” needs to take noise into account



Maximum Likelihood (ML) Examples

Prob(Data | Image) = $\boldsymbol{L(y, \bar{y})}$ (*likelihood*)

$\boldsymbol{y}$: measured data; $\boldsymbol{\bar{y}}$: estimated data (depends on the image)

- Normal distribution

$$\log L\,(y, \bar{y}) = -\sum_b \frac{(y_b - \bar{y}_b)^2}{2{\sigma_b}^2} + cst$$

In this case, ML = Weighted Least Squares fitting

cst: terms that do not depend on the image

- Poisson distribution

$$\log L\,(y, \bar{y}) = \sum_b y_b \log \bar{y}_b - \bar{y}_b + cst$$

MLEM: Maximum Likelihood via Expectation Maximisation

- A “standard” iterative algorithm for Maximum Likelihood (ML) estimation (for Poisson data)
- It “converges” to the ML solution.
- It involves forward and back projection, and compares measured and estimated data by division.

UCL

MLEM reconstruction

$$\text{new\_estimate} = \text{current\_estimate} \cdot BP\left[\frac{\text{measured\_data}}{FP\left[\text{current\_estimate}\right]}\right] / BP[1]$$

$$FP = \text{forward\_project}$$

$$FP\left[\text{current\_estimate}\right] = \text{estimated\_data}$$

$$BP = \text{back\_project}$$

Relation to maximising the Poisson log-likelihood

$$new \quad = current \quad + \frac{current}{BP[1]} . BP[\frac{measured}{FP[current]} - 1]$$

Rewrite in terms of the gradient of the Poisson log-likelihood

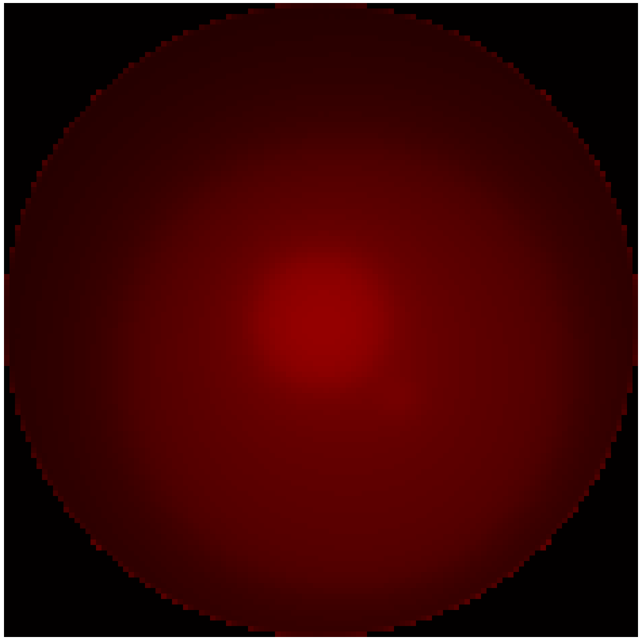
$$new \quad = current \quad + \frac{current}{BP[1]} . \boldsymbol{\nabla L}$$

MLEM Evolution over initial iterations

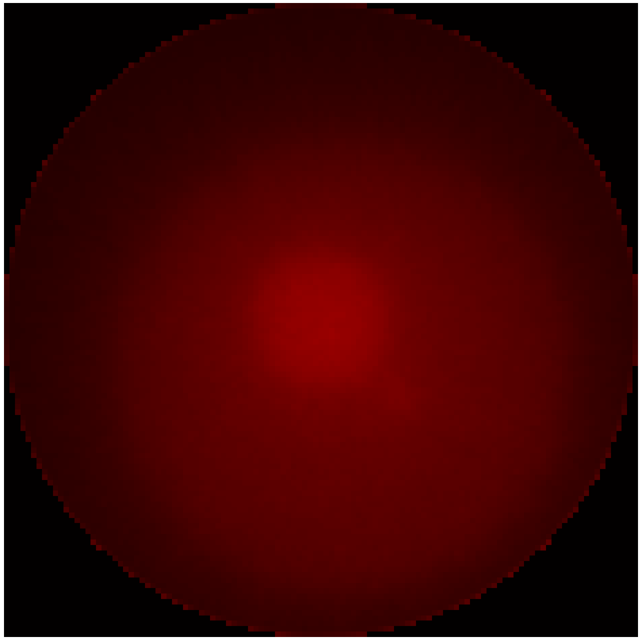
Start with uniform image

Display images in terms of iteration number (1 - 50)

no noise



with noise

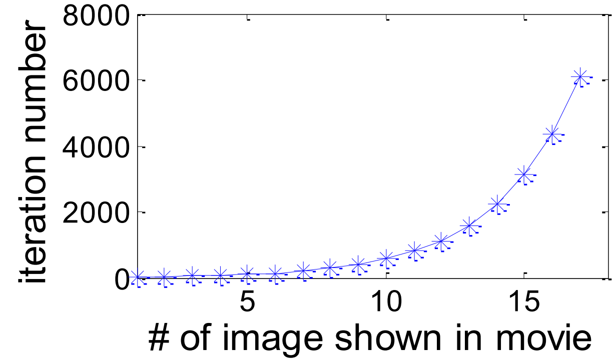


MLEM Evolution over later iterations

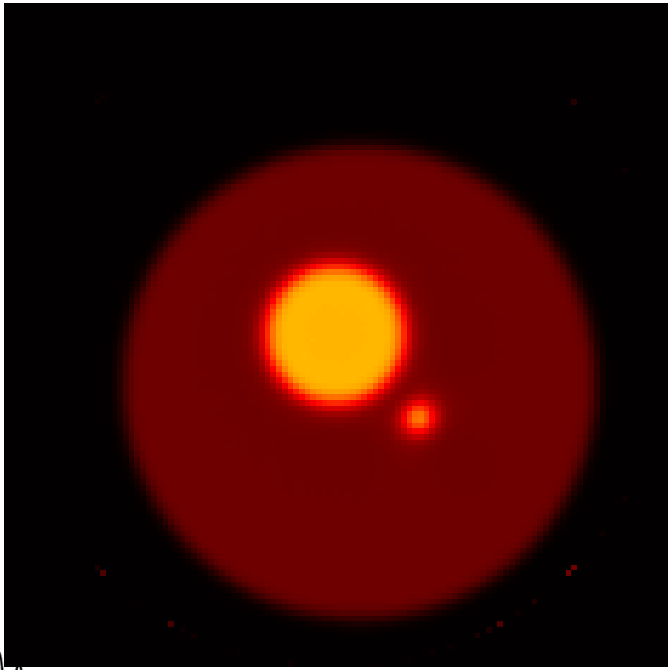
Display images in terms of

log(iteration_number)

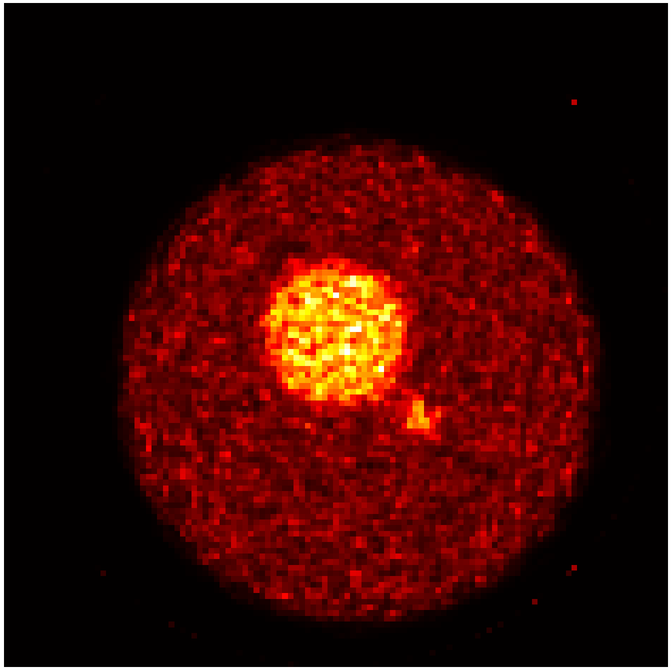
(because EMML convergence slows down)



no noise

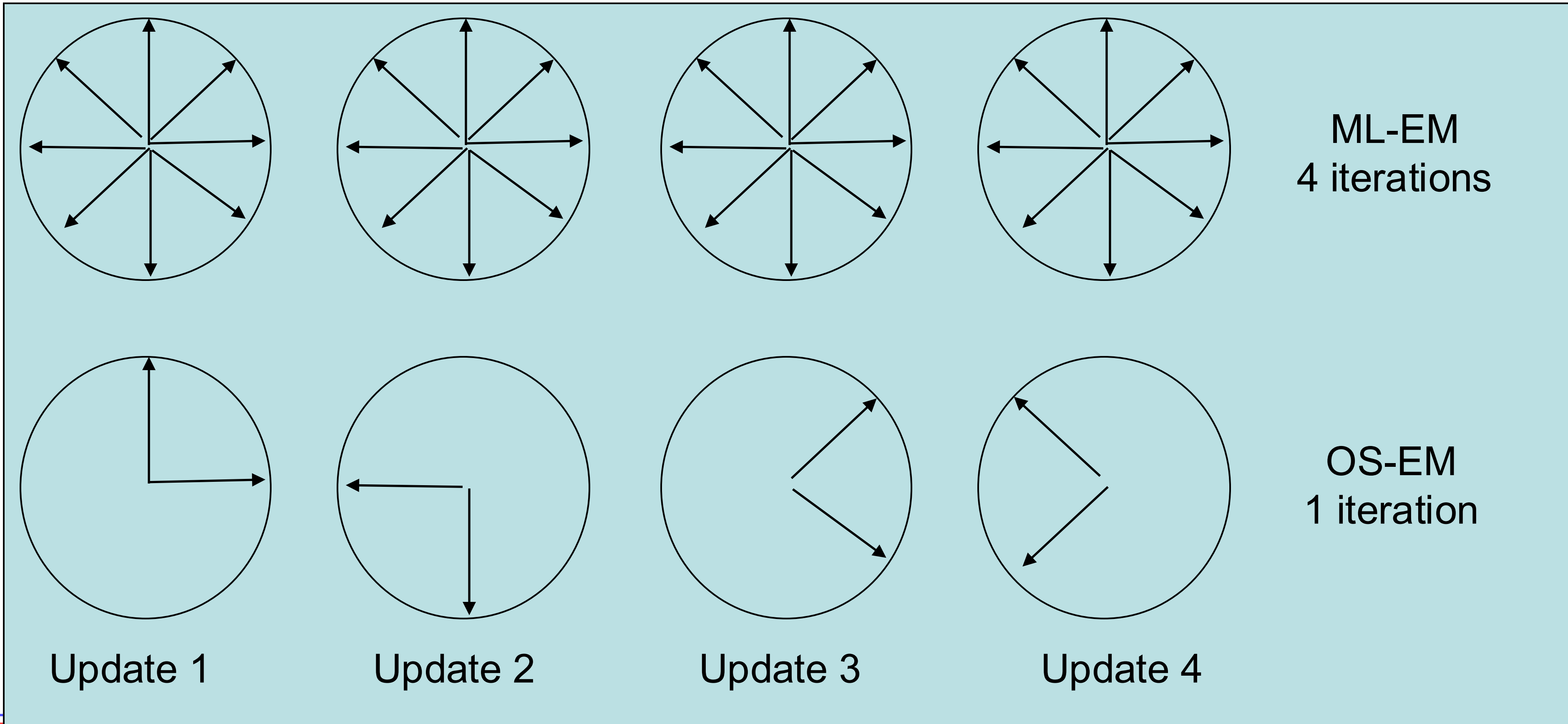


with noise



Acceleration: Ordered Subsets

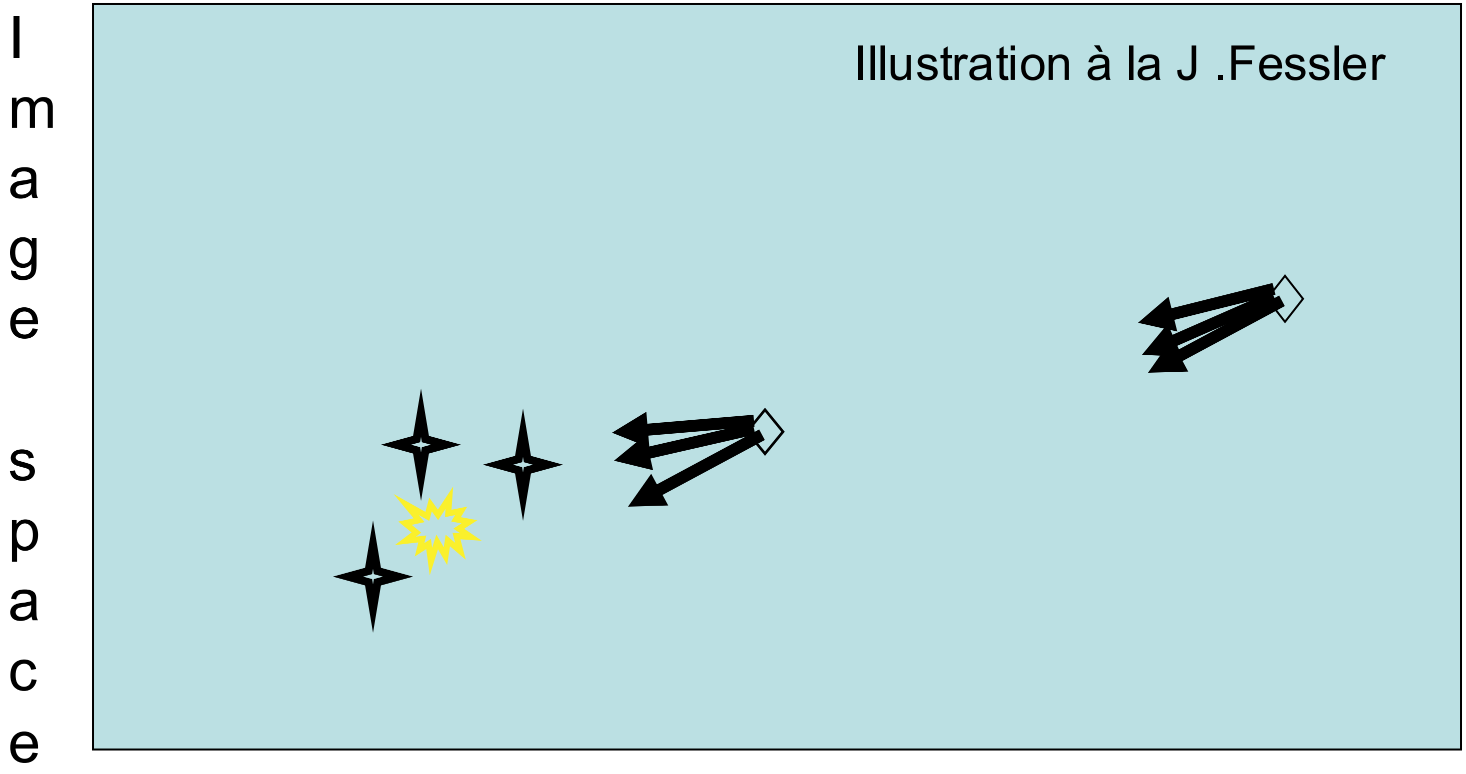
ML-EM: **each update** involves BP and FP for **all** projection angles
OSEM: **each update** only uses **a subset** of projection angles



Much less computation per image update

OSEM: why does it work?

For information only



Note that OSEM does not converge to the ML solution
(worse for noisier data)

but it is never used at high iteration number anyway