Image registration vs image reconstruction

Kris Thielemans
University College London

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Motion estimation

Estimate motion from

- gated PET images
- Gated/dynamic CT or MR images
- both

Use “prior” information

- Rigid or non-rigid (smooth/sliding)
- Quasi-periodic or not
- Motion models

Image registration (single gate)

Minimise objective function

$$L(\alpha) = \Psi(\lambda_t, T(\lambda; \alpha), \alpha)$$

Usually

$$\Psi(\lambda, \hat{\lambda}, \alpha) = S(\lambda_t, \hat{\lambda}) + P(\alpha)$$

with

- λ the target image
- α the deformation field parameters
- $\hat{\lambda} = T(\lambda; \alpha)$ the transformed and resampled image.
- $S(.,.)$ a similarity metric
- $P(.)$ a penalty on the deformation

Connection with image reconstruction:

$$S(y, Fx) + P(x)$$

Transformation/Resampling

The resampling operation is usually split into the following steps

1. Compute "dense" deformation field for α using the deformation field model D (e.g. B-splines)
$$d = D(\alpha)$$
SIRF: **`Transformation`**
note: currently not an "**`operator`**"
2. Resample image (normally via pull-interpolation)
$$\hat{\lambda} = R(\lambda, d)$$
SIRF: **`Resample`**
3. Optional extra step:
Weight the image for density changes (computed by using the Jacobian-determinant of the deformation field)
(could be incorporated into "resampling")

Image registration (gradient w.r.t. α)

$$\frac{\partial L}{\partial \alpha} = \frac{\partial S}{\partial \hat{\lambda}} \frac{\partial \hat{\lambda}}{\partial \alpha} + \frac{\partial P}{\partial \alpha}$$

These are “Jacobian matrices”. Hence need to think about order of multiplications

$$f : \mathbb{R}^n \rightarrow \mathbb{R}^m$$

$$J = [\frac{\delta(f)}{\delta_{(x_1)}} \cdots \frac{\delta(f)}{\delta_{(x_n)}}] = \begin{bmatrix} \frac{\delta f_1}{\delta x_1} & \cdots & \frac{\delta f_1}{\delta x_n} \\ \vdots & \ddots & \vdots \\ \frac{\delta f_m}{\delta x_1} & \cdots & \frac{\delta f_m}{\delta x_n} \end{bmatrix}$$

$m \times n$ matrix

“gradient” is usually reserved for functions, i.e. $m = 1$. Then J is $1 \times n$

We usually write variables as column vectors ($n \times 1$), so will need to transpose

Some observations

- The forward model is non-linear (actually non-affine)
- The optimisation problem is non-convex
- The optimisation problem has local minima

How to handle this?

- Some heuristics (“overlap Centre-of-mass”)
- Multi-resolution pyramid
- Initialise with rigid transformation

Computation of Jacobian determinant related to transformation/registration

$$\frac{\partial\hat{\lambda}}{\partial\alpha} = \frac{\partial\hat{\lambda}}{\partial d}\frac{\partial d}{\partial\alpha}$$

Therefore, we need to be able to compute

- $\tilde{d} = \lambda_2^t \frac{\partial\widehat{\lambda}}{\partial d};$

```
R.multiply_Jacobian_det_wrt_DVF_with_image(lambda2)
```

- $\tilde{\alpha} = \widetilde{d}^t \frac{\partial d}{\partial\alpha};$

- ```
  D.multiply_Jacobian_det_wrt_alpha_with_DVF (dtilde)
  D.adjoint(dtilde)
  ```
  ```

Multiple images (gates)

One "fixed" image, multiple "targets" and deformations

$$L(\boldsymbol{\alpha}) = \sum_g \Psi\left(\lambda_g, T\left(\lambda; \alpha_g\right), \alpha_g\right) + \Pi(\alpha)$$

Motion models have a "model" for $\boldsymbol{\alpha}$. You would then optimise for the model parameters (yet another chain-rule).

Image registration in “acquisition data space”

Non-rigid transformations followed by projections: no longer line integrals

Most people therefore use a “chain”:
first transform the image, then “forward project”

Complication!

In PET the acquisition model depends on the transformation d (through the attenuation map).

Gradients?

We can write this objective function as

$$\Psi(\hat{\lambda};\hat{\mu}) = L(y, F(\hat{\lambda};\hat{\mu}))$$

we need

$$\frac{\partial L}{\partial \alpha} = \left(\frac{\partial \Psi}{\partial d}\right)\frac{\partial d}{\partial \alpha}$$

$$\frac{\partial L}{\partial \alpha} = \left(\frac{\partial \Psi}{\partial \hat{\lambda}}\frac{\partial \hat{\lambda}}{\partial d} + \frac{\partial \Psi}{\partial \hat{\mu}}\frac{\partial \hat{\mu}}{\partial d}\right)\frac{\partial d}{\partial \alpha}$$

“New” ingredients

- $\frac{\partial\Psi}{\partial\hat{\lambda}}$: “traditional” gradient of objective function w.r.t. the emission image (computed at the transformed image)

- $\frac{\partial\Psi}{\partial\hat{\mu}} = \left.\frac{\partial\Psi}{\partial\mu}\right|_{\mu=\hat{\mu}}$, i.e. gradient of objective function w.r.t. attenuation image (computed at the transformed image).

Note that $\frac{\partial\Psi}{\partial\mu} = \frac{\partial\hat{y}}{\partial\mu}\frac{\partial\Psi}{\partial\hat{y}}$`

(Forward) acquisition model with motion

Gated

$$\hat{y}_g(\lambda) = e^{-R\mu_g} A\lambda_g + r_g = F(\lambda_g; \mu_g)$$

Motion

$$\lambda_g = W_g\lambda,\ \mu_g = W_g\mu$$

Combine

$$\hat{y}_g(\lambda) = e^{-RW_g\mu} AW_g\lambda + r_g = F(W_g; \mu_g)$$

Objective function with motion

- Single gate

$$L_g(\lambda) = L(y_g, \hat{y}_g(\lambda))$$

with

$$\hat{y}_g(\lambda) = F(W_g\lambda; \mu_g)$$

- Gradient w.r.t. λ

$$\frac{\partial L_g}{\partial \lambda} = \frac{\partial \lambda_g}{\partial \lambda}\frac{\partial L}{\partial \lambda_g} = W_g^T \frac{\partial L}{\partial \lambda_g}$$

- All gates

$$L(\lambda) = \sum_g L_g(\lambda)$$

Gradient of objective function w.r.t. μ

Recall that $\hat{y} = e^{-R\mu}A\lambda + r$

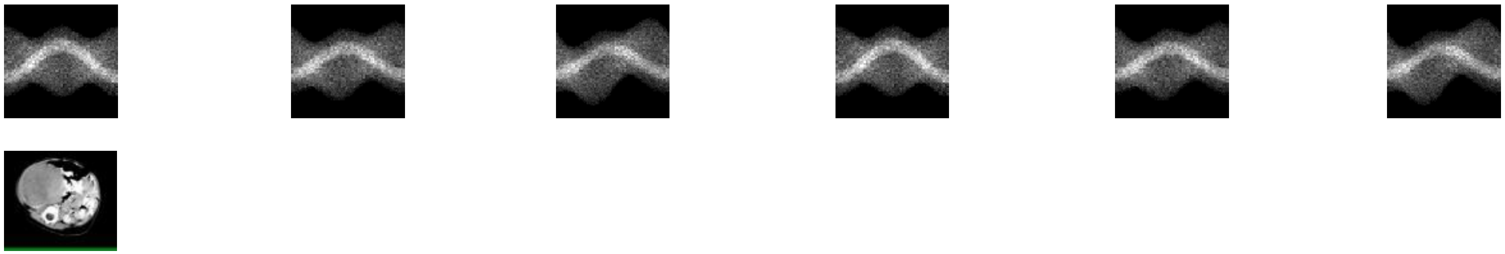
Therefore

$$\frac{\partial \hat{y}_b}{\partial \mu_v} = -R_{bv}(e^{-R\mu}A\lambda)_b = -R_{bv}(\hat{y} - r)_b$$

So, $\left(\frac{\partial \hat{y}}{\partial \mu}\tilde{y}\right)_v = \sum_b \frac{\partial \hat{y}_b}{\partial \mu_v}\tilde{y}_b = -R^t[\tilde{y}.(\hat{y} - r)]$

Note that for Poisson, we need this for $\tilde{y} = \frac{\partial \Psi}{\partial \hat{y}} = -(\frac{y}{\hat{y}} - 1)$. This needs leads to some nice cancelations if $r = 0$, but it generally isn't.

Joint motion estimation/PET image reconstruction: single CT (or MR) image



$$\bar{g}(f, W_g, \mu) = e^{-RW_g\mu} P W_g f + r_g$$

$$\max_{f, W_g} \sum_g L\left(y_g, \bar{y}(f, W_g, \mu)\right)$$

Deformation $\tilde{\mu} = \widetilde{W}\mu$ as input?
Estimated $W_g f$ and $W_g \tilde{\mu}$ are identical!

Bousse al, Maximum-Likelihood Joint Image Reconstruction/Motion Estimation in Attenuation-Corrected Respiratory Gated PET/CT using a Single Attenuation Map, IEEE TMI 2015

How to solve the joint problem?

Option 1: the "optimist"

- Consider a new set of variables $x = [\lambda, \boldsymbol{\alpha}]$
- Throw it into CIL!

Option 2: "grouped coordinate descent":

Alternate between

- Find λ given α (MCIR)
- Find α given λ (sinogram-based image registration)

Conclusion and outlook

- We only need a few extra methods in SIRF to be able to write our own
  - Image-based registration
  - Acquisition-data-based registration
  - Joint motion and image estimation
- There will then be many methods that can be implemented, and a lot of evaluation will be needed.
- Extensions to kinetics are possible but require (even) more computational efforts.